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COMMON SCHOOL ARITHMETIC

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COMMON SCHOOL ARITHMETIC

BY

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PREFACE.

ARITHMETIC is probably one of the most poorly taught of the common school branches. The texts in common use are chiefly at fault. They burden the learner with so great a number of minor details and useless applications as to render it impossible for him to unify the lesser truths about the greater. Thus the pupil fails not only of a competent practical knowledge of the subject, but also of the highest discipline which is the outcome of mastery alone. The method, therefore, adopted in this treatise is the *Intensive.* Clearness, however, has in no case been sacrificed to conciseness; but, on the contrary, it is believed that the treatment in general, and the demonstrations in particular, will be found to be not simply brief but at the same time pointed and luminous.

The Vertebral Principle of the book is Ratio. This, the Newtonian conception of number, not only imparts a new dignity and importance to Proportion as making it sustain the same relation to Arithmetic that the Equation does to Algebra, but also integrates the several membral divisions of Arithmetic into the strictest unity. The book will therefore be found to be easy of thorough mastery — the first desideratum in a text.

The presentation of Fractions in the Concrete (Chapter from the Primary Arithmetic) as *integers* of lower orders of units will make the transition from the integral to the fractional idea easy and natural. This method of introducing fractions proper is altogether natural. The newsboy "handles" coppers, nickels, dimes, quarters, and halves with as little difficulty as the grocer measures syrup or vinegar by the gill, pint, and quart — and all the while unconscious of dealing with fractions.

BOTHWELL GRAHAM.

ATHENS, GA., 1895.

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COMMON SCHOOL ARITHMETIC.

CHAPTER I.

NOTATION AND NUMERATION.

Art. **1.** **Quantity** is the amount or extent of anything.

2. A **Unit** is a standard in terms of which a quantity is measured. A unit is one; that is, one time or once the standard of measure.

To measure the length of a floor in terms of the yardstick, it would be necessary only to find how many times the standard of measure could be applied to the longer side of the floor. Suppose it to be applied five times, then the length of the floor is five yards. The unit in this case is not one yard, but *once*, or one time a yard; and the number is not five *yards*, but *five*, or five *times a* yard.

3. A **Number** is the measure of a quantity in terms of a unit, natural or assumed. It is the "how many" *times*, and expresses merely the relation, or ratio, which one quantity bears to another of the same kind used as the unit, or standard of measure. In 5 apples, 3 quarts, 5 and 3 are numbers. The unit in the first example is *one* apple, — a natural unit; in the second, *one* quart, — an assumed or conventional unit.

More accurately, the units are *once*, or *one time* an apple, *once*, or *one time* a quart. Conversely, 1 apple, 1 quart, may be measured in terms of 5 apples, 3 quarts; and the comparison will yield the numbers $\frac{1}{5}$, $\frac{1}{3}$, known as reciprocals, respectively, of 5 and 3. They are likewise distinguished, the latter as *whole* numbers, or *Integers;* the former, as *Fractions.* These two ideas, the integral and its fractional reciprocal, are both fundamentally involved in every number as converse terms of the same relation. *An* apple becomes *one* apple only through an implied reference to two or more apples; and a collection of apples, say 5 apples, becomes such only through an implied reference to *one* apple. That is, the fundamental idea in number is ratio. This appears clearly in the names of fractional numbers; as, one-half, one-third, four-fifths, etc.

4. But while a number primarily signifies ratio or comparison, it may secondarily imply aggregation or grouping; as, 5 grapes suggests a cluster of 5 grapes. Hence, in a secondary sense, a number is a unit or a collection of units.

[This definition implicitly contemplates the group as a unit of a higher order.]

5. For convenience larger quantities are usually measured in terms of two or more units of different orders which bear a fixed and constant relation to each other. Such numbers are properly called **Compound.** Nineteen quarts is more readily thought in terms of two orders of units; as, 4 gallons, 3 quarts. Twenty-five is two tens, five.

6. A number whose kind of unit is expressed, is said to be **Concrete;** as, 2 boys, 8 sheep, 13 oranges.

7. A number whose kind of unit is not expressed, is said to be **Abstract;** as, 3, 7, 10.

8. A number expressed in related concrete units of different orders is called **Compound Denominate**; as, 3 shillings, 4 pence, 2 farthings; 3 weeks, 5 days, 23 hours.

9. Numbers may be increased, diminished, repeated, partitioned; hence arise Addition, Subtraction, Multiplication, Division, — the fundamental operations of Arithmetic.

10. To simplify computation, symbols are used to represent numbers, and signs to represent the operations to be performed on them.

The method of expressing numbers in symbols is called **Notation**; and that of reading them, when thus expressed, **Numeration.**

There are two systems of Notation in common use, — the *Arabic,* which uses figures, and the *Roman,* which uses letters.

11. The Arabic employs ten characters, or figures.

They are:

0	zero. cipher. naught.	2,	two.	6,	six.
		3,	three.	7,	seven.
		4,	four.	8,	eight.
1,	one.	5,	five.	9,	nine.

12. These are the only abstract numbers really simple; that is, that are thought in the *single primary* unit and expressed in a *single figure.* Larger numbers are thought in their several orders of units, and are expressed in a corresponding number of figures, so placed as to indicate the order of these units.

TABLE OF RELATIONS.

13. Ten Units make a Ten (Unit of second order).
Ten Tens make a Hundred (Unit of third order).
Ten Hundreds make a Thousand (Unit of fourth order).
Etc. Etc.

14. LAW OF PLACE. — *Each figure occupies that place, counting from the right, that corresponds in number with the order of its unit.*

15. The simple value of a figure is its value in simple units, without reference to its ordinal value.

16. The local value of a figure is that ordinal value which it acquires through the operation of the following

GENERAL PRINCIPLE. — *The value of a figure is multiplied by ten by each removal one place to the left, and divided by ten by each removal one place to the right.*

17. Zero, or naught, has no value, and is simply used to denote the absence of an order, or orders, of units; as, 500 denotes 5 units of 3d order, no units of 2d, no units of 1st.

18. TABLE ILLUSTRATING THE ARABIC SYSTEM OF NOTATION AND NUMERATION.

PERIODS NUMBER.	8th.	7th.	6th.	5th.	4th.	3d.	2d.	1st.
PERIODS NAMES.	Sextillions	Quintillions	Quadrillions	Trillions	Billions	Millions	Thousands	Units
ORDERS.	Hundreds Tens Units	Hundreds Tens Units	Hundreds Tens Units	Hundreds Tens Units	Hundreds Tens Units	Hundreds Tens Units	Hundreds Tens Units	Hundreds Tens Units
	9 7 5	3 1 0	2 4 6	8 1 3	7 5 6	0 7 3	4 0 0	6 4 2

This number is read nine hundred seventy-five sextillions, three hundred ten quintillions, two hundred forty-six quadrillions, eight hundred thirteen trillions, seven hundred fifty-six billions, seventy-three millions, four hundred thousands, six hundred forty-two.

19. The table may be extended from sextillions to septillions, octillions, nonillions, decillions, etc.; but these higher orders of units have little or no practical utility.

Note. — The pupil should name the periods in order, forward and backward, until the sequence in both directions is firmly fixed in the mind.

Rule for Notation.

20. I. *Begin at the left hand and write the figures belonging to the highest period.*

II. *Write the figures of each successive period in its order.*

Rule for Numeration.

21. *Beginning at the right hand, separate the number into periods of three figures each. From the left hand, read each period separately, giving the name of each period except the last.*

22. Read the following in terms of their several orders of units, and then in terms of their primary units:

56
75
34
43
35
57
73

80
93
19
86
89
27
32

100
127
156
173
137
153
276

333
341
567
765
892
953
999

23. Prepare the following numbers for reading by copying and pointing off into periods; then read them:

5130
7242
6402
8200
9041
7643
3005
5073
9683

87642
93001
90034
85371
97531
86420
78563
82756
98766

565325
743644
999999
876543
123456
100001
173968
194763
345543

65304225
72643756
90000000
95325635
87246731
89400301
76434728
99999999
88333555
86021346
79489879
83467057

1927623704
9642013579
8430004764
9643789425
6789345578
7000000000
8000000003
7135796845
6478967837
94787245678
36842463750
19876542100

501413030400300200001
430000000000000001010
222200020002990009999
384792840004678789421
764594378945768943007
996753301472689007843

24. Express in the Arabic notation:

1. Five hundred six.
2. Seven hundred nineteen.
3. Forty-seven thousand, sixteen.
4. Ninety-four thousand, seven hundred thirteen.
5. Six hundred thousand, seventy-five.
6. Nine hundred thousand, six hundred forty-three.
7. 972 thousand, 743.
8. Four hundred sixty-five thousand, three hundred thirty-five.
9. Eight hundred thousand, eight.
10. Thirty-three million, thirty-three thousand, thirty-three.
11. Three hundred million, thirty thousand, three.
12. Five hundred sixty-seven million, seven hundred eighty-four thousand, six hundred ninety-five.
13. Five billion, fifty million, five hundred thousand, five.
14. Four hundred twenty billion, three hundred sixty-one million, one hundred eleven thousand, nine hundred eighty-seven.
15. Six hundred trillion, seven hundred forty-three million, four hundred seventy-five thousand, one hundred thirty-three.
16. Seven hundred twenty-nine trillion, six hundred thirteen billion, twelve million, two thousand, one.
17. Eight hundred eighty-eight trillion, six hundred sixty-six billion, five hundred fifty-five million, four hundred forty-four thousand, three hundred thirty-three.
18. Nine trillion, ninety billion, nine hundred million.

19. Five hundred fifty-five trillion, fifty-five million, five.

20. 771 trillion, 842 million, 964.

21. 35 billion, 47 thousand.

22. 67 trillion, 63.

23. 87 trillion, 3 billion, 50 million, 200 thousand, 6.

24. 11 thousand, 11 hundred 11.

25. 12 thousand, 12 hundred 12.

26. 18 thousand, 18 hundred 18.

25. The Roman notation employs these seven capital letters:

I = 1, L = 50,
V = 5, C = 100,
X = 10, D = 500,
M = 1000.

All numbers may be expressed by combining these seven letters according to the following laws:

I. *Repeating a letter repeats its value.*

II. *When a letter of less value follows one of greater value, its value is added to that of the greater.*

III. *When a letter of less value precedes one of greater value, its value is subtracted from that of the greater.*

IV. *When a letter of less value comes between two letters each of greater value, its value is subtracted from that of the latter, and the result added to the former.*

V. *A bar placed over a letter multiplies its value by* 1000.

II = 2.
III = 3.
IV = 4.
VI = 6.
VII = 7.
VIII = 8.
IX = 9.

XIV = 14.
XIX = 19.
XVI = 16.
XXX = what?
XL = "
XC = "
CCC = "

DC = what?
MDC = "
$\overline{\text{X}}$ = "
MDCC = "
MDCCCXCV = "
$\overline{\text{M}}$ = "
$\overline{\text{MDC}}$ = "

CHAPTER II.

ADDITION AND SUBTRACTION.

26. **Addition** is the counting together of two or more numbers. **Subtraction** is the counting off of one number from another.

Or, **Addition** is the process of finding a single number that is equal to two or more given numbers. The numbers added are called **Addends**; the result obtained, the **Sum.**

27. **Subtraction** is the process of finding the difference between two numbers. The number to be diminished is called the **Minuend**; that to be subtracted, the **Subtrahend**; and the result, the **Difference,** or **Remainder.**

28. Addition and subtraction are converse processes, and each process may be verified by the other.

29. The sign of addition is a perpendicular cross, written thus, +, and called **Plus** (more, or increased by).

30. The sign of subtraction is a horizontal line, written thus, –, and called **Minus** (less). $5 + 3 = 8$ is read, "5 plus 3 equals 8." $8 - 3 = 5$ is read, "8 less 3 (or 8 diminished by 3) equals 5."

31. Find the sum of 48 and 35.

OPERATION (*a*).	OPERATION (*b*).
48	48
35	35
13	83
7	
83	

Explanation. —1. Write the addends so that units of the same order shall fall in the same vertical column.

2. Begin at the right and add the figures of the first column, thus, 5, 13; 13 is 1 ten and 3 units. Write the 3 units under the first or units' column, and the 1 ten under the second column (as in operation *a*); or "carry" the 1 ten and add to the lower figure of the second column (as in operation *b*).

3. (In operation *a*.) Add the figures of the second column, thus, 3, 7. Write the 7 tens under the 1 ten. The sum is 1 ten and 7 tens (8 tens) 3 units, 83.

(In operation *b*.) Add the second column thus, 1 (carried) 4, 8. Write the 8 tens under the second column.

4. The sum of 48 and 35 is therefore 83.

32. RULE FOR ADDITION.

I. *Write the addends so that units of the same order shall stand in the same column.*

II. *Begin with the units' column and add each column separately. If its sum is less than ten, write it under the column added.*

III. *If the sum of any column is ten or more, write the first, or right-hand figure, only, and "carry" the other figure, or figures, to the next column.*

IV. *Write the sum of the last column.*

33. PROOF. — Subtract either addend from the sum; the result will be the other addend(s).

34. Find the difference between 83 and 48.

OPERATION.

$$\begin{array}{r@{\;=\;}l}
83 & 70 + 13 \\
\underline{48} & \underline{40 + 8} \\
35 & 30 + 5
\end{array}$$

EXPLANATION. — 1. Write the less number under the greater, placing units under units, tens under tens, etc.

2. Begin at the right hand to subtract.

3. As 8 units cannot be taken from 3 units, "borrow" 1 ten, which equals ten units, from 8 tens; 10 units and 3 units = 13 units; 13 units less 8 units = 5 units: write the 5 units in the units' place.

4. 7 tens less 4 tens = 3 tens, which is written in the tens' place.

5. Therefore the difference between 83 and 48 is 35.

35. RULE FOR SUBTRACTION.

I. *Write the subtrahend under the minuend, units under units, tens under tens, etc.*

II. *Begin at the right hand and subtract each figure in the subtrahend from the one above, if it can be done, and set the remainder underneath.*

III. *If any figure in the subtrahend is greater than the figure above, add ten to the figure of the minuend before subtracting, and make the next figure of the minuend one less.*

36. PROOF.—Add the difference to the subtrahend; the sum will equal the minuend.

37. Examples for Practice.

1. 376 + 459 + 378 + 294 + 168 + 397.
2. 212 + 319 + 173 + 896 + 742 + 325.
3. 376 + 249 + 596 + 637 + 892 + 675.
4. 945 − 753; 684 − 486; 733 − 335.
5. 1295 + 3475 + 6942 + 8911 + 3377.
6. 3245 + 6789 − 3424 + 6459 + 7243.
7. 5964 + 8972 − 7846 + 8976 − 9673.
8. 1124 − 836; 7259 − 5468; 8743 − 5371.
9. 1678 − 943; 1600 − 978; 9204 − 3879.
10. 7359 + 6742 − 9872; 8430 − 6457.

11.	12.
1653041	47296432
8903714	27538976
8761043	27650005
1439177	84579853
9346825	35246782
4122581	96578930
1055837	45678910
2464123	32146835

13. 8794 − 7451; 9857 − 4507.

14. 892 − 283; 800700 − 275541.

15. 987654321 − 123456789.

16. 1000000 − 1; 1000000 − 999999.

17. 1000000 − 56409; 999888209 − 444555902.

Find the sum of the following:

18.	**19.**	**20.**
1597543	3764297	47333268
2057893	4636729	75431942
2467820	8412345	94256873
1357864	6789047	14217116
2789437	9876543	91864057
8426789	2100047	58673245
5738659	9653042	13436423
5378247	8756453	56328947

21.	**22.**
24463154	347896721
21757854	892136789
12349234	372184796
54264392	874347654
76332456	724645974
96427345	964735851
97646201	872954592
73564572	333472964
89346792	759893846
89464729	693744872
93483035	943784345
39400287	871672593

23.	**24.**	**25.**
824673124	942678943927	468937
764371942	125724326278	541272
567893741	842126427832	486573
964789421	872446502163	724237
731642583	426732088586	427843
583729647	754234923854	789264
781964592	455587527621	342844
892789300	724326832987	427768
946207351	437523431627	342145
742653001	542634853921	273642
165734221	214875246736	898432
724567250	441457637542	245389
934576402	234426732892	252454
874567876	637230242757	932481

26.	**27.**
543781659	459673421
731954643	896427871
789345672	764593479
245678924	876425731
934576472	407087345

28. 52673052 – 40658213.

29. 123614208 – 42397406.

30. 328347675 – 5536076.

31. 104202196 – 8592678.

32. 186230410075 – 8599190074.

33. 637230242757 – 23442673282.

34. 934576402 – 898432.

35. 5772943843 – 24439997.

36. 942678943927 – 824673124.

CHAPTER III.

MULTIPLICATION AND DIVISION.

38. **Multiplication** is the process of taking one number as many times as there are units in another.

39. **Division** is the process of taking one number as many times as there are units in the *reciprocal* of another.

40. The **Multiplicand** is the number multiplied.

41. The **Multiplier** is the number multiplied by.

42. The **Product** is the result obtained by the multiplication.

43. The multiplier and multiplicand are called **Factors** of the product.

44. The **Dividend** is the number divided.

45. The **Divisor** is the number divided by.

46. The **Quotient** is the result obtained by division.

47. The **Remainder** is the part of the dividend left undivided.

48. The **Sign of Multiplication** is the oblique cross [×]. It is read "*times*" or "*multiplied by.*" Thus, 3×5 is read "3 times 5," or "3 multiplied by 5," according as 3 is considered the multiplier or the multiplicand.

49. The **Sign of Division** is a horizontal line with a dot above and a dot below [÷]. It shows that the number that precedes it is to be divided by the number that follows it. Thus, $40 \div 4$ is read "40 divided by 4."

50. Division is also indicated by writing the dividend above a line and the divisor below it. Thus, $\frac{40}{4}$ shows that 40 is to be divided by 4. It may also be written $\frac{1}{4}$ of 40.

51. Multiplication and division are the reciprocal of each other, in the same sense that an integer is the reciprocal of a fraction.

For in measuring a quantity in terms of a standard, which process gives rise to number, the unit is repeated, taken a given number of times; that is, *multiplied.*

Let the result obtained be 3. Then the quantity measured stands to the unit as 3 to 1, or it is 3 times the unit.

Conversely, when the unit is measured in terms of the given quantity, the result $\frac{1}{3}$ is obtained. That is, an integer is a product and its reciprocal fraction a quotient.

As in repeating a unit of measure we multiply it, so in multiplying a number we repeat it as a higher unit.

To multiply 5 by 3, we take 5 three times, which yields the result three 5's. And this expressed in primary units is 15; but the true product is three 5's, and not its reduced form 15. The reduction is accomplished by adding three 5's together; whence some have defined multiplication to be a short method of addition of equal numbers.

In the true product (three 5's), 5 is the unit or multiplicand, and 3 is the multiplier. Here we have measured a quantity in terms of 5, and since we have taken the unit 3 times, we have multiplied it by 3.

The converse process, to measure the unit 5 in terms of the product, three 5's, gives as the quotient $\frac{1}{3}$ of three 5's. Hence, to multiply by 3, the quantity multiplied is taken 3 times; to divide by 3, the quantity divided is taken $\frac{1}{3}$ time, or is multiplied by $\frac{1}{3}$.

To the Teacher. — Take special pains that the pupil may get a clear notion of the reciprocal. To this end, fully illustrate it by examples; as, a yard measured in terms of a foot gives 3, an integer; and conversely a foot measured in terms of a yard gives $\frac{1}{3}$, a fraction. Bring the pupil to see that an integer and the reciprocal fraction are simply converse terms of the same relation, that we cannot recognize the relation of 3 to 1, without being, at the same time, conscious of the relation of 1 to 3. It will then be easy to lead him to see that multiplication and division are, also, in the same sense, converse faces of the same idea.

52. To take a quantity a part of a time is to take a part of the quantity; therefore, division may be defined as the process of separating a quantity into a number of equal parts. Or, since a number may be separated, or partitioned, into a number of equal parts by finding how many times the number will contain the number of parts, division may be defined to be the process of finding how many times one number is contained in another.

EXAMPLE. — Let it be required to separate 12 apples into 4 equal parts or parcels. Each parcel will get 1 apple of every 4 apples; therefore, in each parcel there will be as many times *one* apple as 4 apples is contained times in 12 apples, which is 3. That is, 1 of the 4 equal parts of 12 apples is 3 apples.

To find how often one number is contained in another is properly nothing but a reduction. 12 is twelve 1's, and to find how many 4's are in 12 is to express 12 in terms of 4; 12 is three 4's.

53. The three methods of division may conveniently be distinguished as Division Proper (converse multiplication), Partitive Division (partitioning), and Reductive Division.

TO THE TEACHER. — The actual process of division may, perhaps, be more readily learned when it is explained in terms of the reductive method; but its practical applications become much more difficult by reason of the unnecessary circumlocution introduced into the logical procedure. If you are skeptical, require your class to divide 12 apples among 4 boys by the reductive method, and then ask, what is your divisor, 4 boys or 4 apples? Why? What is your quotient, 3 apples or 3 (abstract)? If your quotient is abstract, how do you obtain 3 *apples* as each boy's share? The process of each method will be explained, and the teacher may adopt the one of his choice.

54. Multiplication and Division Table.

$2 \times 1 = 2$	$\frac{1}{2} \times 2 = 1$	$3 \times 1 = 3$	$\frac{1}{3} \times 3 = 1$
$2 \times 2 = 4$	$\frac{1}{2} \times 4 = 2$	$3 \times 2 = 6$	$\frac{1}{3} \times 6 = 2$
$2 \times 3 = 6$	$\frac{1}{2} \times 6 = 3$	$3 \times 3 = 9$	$\frac{1}{3} \times 9 = 3$
$2 \times 4 = 8$	$\frac{1}{2} \times 8 = 4$	$3 \times 4 = 12$	$\frac{1}{3} \times 12 = 4$
$2 \times 5 = 10$	$\frac{1}{2} \times 10 = 5$	$3 \times 5 = 15$	$\frac{1}{3} \times 15 = 5$
$2 \times 6 = 12$	$\frac{1}{2} \times 12 = 6$	$3 \times 6 = 18$	$\frac{1}{3} \times 18 = 6$
$2 \times 7 = 14$	$\frac{1}{2} \times 14 = 7$	$3 \times 7 = 21$	$\frac{1}{3} \times 21 = 7$
$2 \times 8 = 16$	$\frac{1}{2} \times 16 = 8$	$3 \times 8 = 24$	$\frac{1}{3} \times 24 = 8$
$2 \times 9 = 18$	$\frac{1}{2} \times 18 = 9$	$3 \times 9 = 27$	$\frac{1}{3} \times 27 = 9$
$2 \times 10 = 20$	$\frac{1}{2} \times 20 = 10$	$3 \times 10 = 30$	$\frac{1}{3} \times 30 = 10$
$2 \times 11 = 22$	$\frac{1}{2} \times 22 = 11$	$3 \times 11 = 33$	$\frac{1}{3} \times 33 = 11$
$2 \times 12 = 24$	$\frac{1}{2} \times 24 = 12$	$3 \times 12 = 36$	$\frac{1}{3} \times 36 = 12$

$4 \times 1 = 4$	$\frac{1}{4} \times 4 = 1$	$5 \times 1 = 5$	$\frac{1}{5} \times 5 = 1$
$4 \times 2 = 8$	$\frac{1}{4} \times 8 = 2$	$5 \times 2 = 10$	$\frac{1}{5} \times 10 = 2$
$4 \times 3 = 12$	$\frac{1}{4} \times 12 = 3$	$5 \times 3 = 15$	$\frac{1}{5} \times 15 = 3$
$4 \times 4 = 16$	$\frac{1}{4} \times 16 = 4$	$5 \times 4 = 20$	$\frac{1}{5} \times 20 = 4$
$4 \times 5 = 20$	$\frac{1}{4} \times 20 = 5$	$5 \times 5 = 25$	$\frac{1}{5} \times 25 = 5$
$4 \times 6 = 24$	$\frac{1}{4} \times 24 = 6$	$5 \times 6 = 30$	$\frac{1}{5} \times 30 = 6$
$4 \times 7 = 28$	$\frac{1}{4} \times 28 = 7$	$5 \times 7 = 35$	$\frac{1}{5} \times 35 = 7$
$4 \times 8 = 32$	$\frac{1}{4} \times 32 = 8$	$5 \times 8 = 40$	$\frac{1}{5} \times 40 = 8$
$4 \times 9 = 36$	$\frac{1}{4} \times 36 = 9$	$5 \times 9 = 45$	$\frac{1}{5} \times 45 = 9$
$4 \times 10 = 40$	$\frac{1}{4} \times 40 = 10$	$5 \times 10 = 50$	$\frac{1}{5} \times 50 = 10$
$4 \times 11 = 44$	$\frac{1}{4} \times 44 = 11$	$5 \times 11 = 55$	$\frac{1}{5} \times 55 = 11$
$4 \times 12 = 48$	$\frac{1}{4} \times 48 = 12$	$5 \times 12 = 60$	$\frac{1}{5} \times 60 = 12$

$6 \times 1 = 6$ $\frac{1}{6} \times 6 = 1$ $7 \times 1 = 7$ $\frac{1}{7} \times 7 = 1$

$6 \times 2 = 12$ $\frac{1}{6} \times 12 = 2$ $7 \times 2 = 14$ $\frac{1}{7} \times 14 = 2$

$6 \times 3 = 18$ $\frac{1}{6} \times 18 = 3$ $7 \times 3 = 21$ $\frac{1}{7} \times 21 = 3$

$6 \times 4 = 24$ $\frac{1}{6} \times 24 = 4$ $7 \times 4 = 28$ $\frac{1}{7} \times 28 = 4$

$6 \times 5 = 30$ $\frac{1}{6} \times 30 = 5$ $7 \times 5 = 35$ $\frac{1}{7} \times 35 = 5$

$6 \times 6 = 36$ $\frac{1}{6} \times 36 = 6$ $7 \times 6 = 42$ $\frac{1}{7} \times 42 = 6$

$6 \times 7 = 42$ $\frac{1}{6} \times 42 = 7$ $7 \times 7 = 49$ $\frac{1}{7} \times 49 = 7$

$6 \times 8 = 48$ $\frac{1}{6} \times 48 = 8$ $7 \times 8 = 56$ $\frac{1}{7} \times 56 = 8$

$6 \times 9 = 54$ $\frac{1}{6} \times 54 = 9$ $7 \times 9 = 63$ $\frac{1}{7} \times 63 = 9$

$6 \times 10 = 60$ $\frac{1}{6} \times 60 = 10$ $7 \times 10 = 70$ $\frac{1}{7} \times 70 = 10$

$6 \times 11 = 66$ $\frac{1}{6} \times 66 = 11$ $7 \times 11 = 77$ $\frac{1}{7} \times 77 = 11$

$6 \times 12 = 72$ $\frac{1}{6} \times 72 = 12$ $7 \times 12 = 84$ $\frac{1}{7} \times 84 = 12$

$8 \times 1 = 8$ $\frac{1}{8} \times 8 = 1$ $9 \times 1 = 9$ $\frac{1}{9} \times 9 = 1$

$8 \times 2 = 16$ $\frac{1}{8} \times 16 = 2$ $9 \times 2 = 18$ $\frac{1}{9} \times 18 = 2$

$8 \times 3 = 24$ $\frac{1}{8} \times 24 = 3$ $9 \times 3 = 27$ $\frac{1}{9} \times 27 = 3$

$8 \times 4 = 32$ $\frac{1}{8} \times 32 = 4$ $9 \times 4 = 36$ $\frac{1}{9} \times 36 = 4$

$8 \times 5 = 40$ $\frac{1}{8} \times 40 = 5$ $9 \times 5 = 45$ $\frac{1}{9} \times 45 = 5$

$8 \times 6 = 48$ $\frac{1}{8} \times 48 = 6$ $9 \times 6 = 54$ $\frac{1}{9} \times 54 = 6$

$8 \times 7 = 56$ $\frac{1}{8} \times 56 = 7$ $9 \times 7 = 63$ $\frac{1}{9} \times 63 = 7$

$8 \times 8 = 64$ $\frac{1}{8} \times 64 = 8$ $9 \times 8 = 72$ $\frac{1}{9} \times 72 = 8$

$8 \times 9 = 72$ $\frac{1}{8} \times 72 = 9$ $9 \times 9 = 81$ $\frac{1}{9} \times 81 = 9$

$8 \times 10 = 80$ $\frac{1}{8} \times 80 = 10$ $9 \times 10 = 90$ $\frac{1}{9} \times 90 = 10$

$8 \times 11 = 88$ $\frac{1}{8} \times 88 = 11$ $9 \times 11 = 99$ $\frac{1}{9} \times 99 = 11$

$8 \times 12 = 96$ $\frac{1}{8} \times 96 = 12$ $9 \times 12 = 108$ $\frac{1}{9} \times 108 = 12$

$10\times 1=10$	$\frac{1}{10}\times 10=1$	$11\times 1=11$	$\frac{1}{11}\times 11=1$
$10\times 2=20$	$\frac{1}{10}\times 20=2$	$11\times 2=22$	$\frac{1}{11}\times 22=2$
$10\times 3=30$	$\frac{1}{10}\times 30=3$	$11\times 3=33$	$\frac{1}{11}\times 33=3$
$10\times 4=40$	$\frac{1}{10}\times 40=4$	$11\times 4=44$	$\frac{1}{11}\times 44=4$
$10\times 5=50$	$\frac{1}{10}\times 50=5$	$11\times 5=55$	$\frac{1}{11}\times 55=5$
$10\times 6=60$	$\frac{1}{10}\times 60=6$	$11\times 6=66$	$\frac{1}{11}\times 66=6$
$10\times 7=70$	$\frac{1}{10}\times 70=7$	$11\times 7=77$	$\frac{1}{11}\times 77=7$
$10\times 8=80$	$\frac{1}{10}\times 80=8$	$11\times 8=88$	$\frac{1}{11}\times 88=8$
$10\times 9=90$	$\frac{1}{10}\times 90=9$	$11\times 9=99$	$\frac{1}{11}\times 99=9$
$10\times 10=100$	$\frac{1}{10}\times 100=10$	$11\times 10=110$	$\frac{1}{11}\times 110=10$
$10\times 11=110$	$\frac{1}{10}\times 110=11$	$11\times 11=121$	$\frac{1}{11}\times 121=11$
$10\times 12=120$	$\frac{1}{10}\times 120=12$	$11\times 12=132$	$\frac{1}{11}\times 132=12$

$12\times 1=12$	$\frac{1}{12}\times 12=1$	$12\times 7=84$	$\frac{1}{12}\times 84=7$
$12\times 2=24$	$\frac{1}{12}\times 24=2$	$12\times 8=96$	$\frac{1}{12}\times 96=8$
$12\times 3=36$	$\frac{1}{12}\times 36=3$	$12\times 9=108$	$\frac{1}{12}\times 108=9$
$12\times 4=48$	$\frac{1}{12}\times 48=4$	$12\times 10=120$	$\frac{1}{12}\times 120=10$
$12\times 5=60$	$\frac{1}{12}\times 60=5$	$12\times 11=132$	$\frac{1}{12}\times 132=11$
$12\times 6=72$	$\frac{1}{12}\times 72=6$	$12\times 12=144$	$\frac{1}{12}\times 144=12$

55. Let it be required to multiply 473 by 25.

OPERATION.

473	multiplicand.
25	multiplier.
2365	product of 5 times the multiplicand.
946	product of 20 times the multiplicand.
11825	product of 25 times the multiplicand.

Explanation. — 1. Write the multiplier under the multiplicand so that the units shall stand under units, tens under tens, etc.

2. Begin at the right to multiply.

3. Five times 3 are 15; 15 is 1 ten 5 units; write the units in the units' place and "carry" the 1 ten to be added to the product of the tens.

4. Five times 7 tens are 35, which with the 1 ten "carried" are 36 tens; 36 tens = 3 hundred 6 tens; write the 6 tens in the tens' place and "carry" the 3 hundred to the product of the hundreds.

5. Five times 4 hundred are 20 hundred, which with the 3 hundred "carried" are 23 hundred; 23 hundred = 2 thousand 3 hundred; write the 3 hundred in the hundreds' place and the 2 thousand in the thousands' place.

6. Multiply by 2 tens, the second figure of the multiplier; 2 tens times 3 = 3 times 2 tens, which is 6 tens; write the 6 tens in the tens' place.

7. Two tens times 7 tens = 14 hundred; 14 hundred = 1 thousand 4 hundred; write the 4 hundred in the hundreds' place and "carry" the 1 thousand to the product of the thousands.

8. Two tens times 4 hundred = 8 thousand, which with the 1 thousand carried are 9 thousand; write the 9 thousand in the thousands' place.

9. Lastly, add the partial products 2365 and 9460, which give the total product, 11825.

10. Hence, the product of 473 multiplied by 25 is 11825.

56. Rule for Multiplication.

I. *Write the multiplier under the multiplicand so that units shall stand under units, tens under tens, etc., and begin at the right to multiply.*

II. *Multiply each figure in the multiplicand in succession by each figure of the multiplier, and write the first figure of each partial product directly under the figure by which you multiply. Carry as in addition.*

III. *Add the partial products, and their sum will be the required product.*

57. Proof. — Since division is the reciprocal of multiplication, the product divided by either factor will give the other factor.

58. Required the quotient of 2365 divided by 5.

OPERATION.

$$\begin{array}{r} 5\underline{)2365} \\ 473 \end{array}$$

EXPLANATION.—1. Write the divisor at the left, and begin at that side to divide.

2. One-fifth of 2 thousand is not an entire thousand; reduce 2 thousand to 20 hundred and add 3 hundred, which gives 23 hundred; one-fifth of 23 hundred is 4 hundred, with a remainder of 3 hundred; write the 4 hundred in the hundreds' place, and add 3 hundred (30 tens) to the 6 tens, which gives 36 tens.

3. One-fifth of 36 tens is 7 tens, with a remainder of 1 ten. Write the 7 tens in the tens' place, and add the 1 ten (10 units) to the 5 units, which gives 15 units; one-fifth of 15 units is 3 units. Write the 3 in the units' place.

If there should be a remainder, write it over the divisor.

Hence the quotient of 2365 divided by 5 is 473.

59. PROOF.—Since division is the converse of multiplication, the quotient multiplied by the divisor, with the remainder added, will give the dividend.

60. When the products and remainders in division are not written, as above, the process is called **Short Division**; when they are written, as below, the process is called **Long Division.**

61. Let it be required to find the quotient of 11825 divided by 25.

OPERATION.

$$\begin{array}{rl} 25)11825(473 & \\ \underline{100} & \\ 182 & \\ \underline{175} & \\ 75 & \\ \underline{75} & \end{array}$$

Explanation. — 1. Write the divisor at the left of the dividend, as above.

2. One twenty-fifth of 118 hundred is 4 hundred, with a remainder of 18 hundred; or 25 is contained in 118 hundred 4 hundred times, with a remainder of 18 hundred. Write the 4 hundred in the hundreds' place in the quotient, at the right of the dividend, and to the remainder, 18 hundred (180 tens), annex the 2 tens of the dividend, making 182 tens.

3. One twenty-fifth of 182 tens is 7 tens, with a remainder 7 tens; or 25 is contained in 182 tens 7 tens times, with a remainder of 7 tens. Write the 7 tens in the tens' place in the quotient, and to the remainder, 7 tens (70 units), annex the 5 units of the dividend, making 75 units.

4. One twenty-fifth of 75 units is 3 units; or 25 is contained in 75 units 3 units times. Write the 3 units in the units' place in the quotient.

Hence the quotient of 11825 ÷ 25 is 473.

62. Rule for Long Division.

I. *Write the divisor at the left of the dividend.*

II. *Divide the fewest left-hand figures of the dividend that will yield an integral quotient, and write the quotient figure thus found at the right of the dividend.*

III. *Multiply the divisor by this quotient figure, subtract the product from the partial dividend, and bring down the next figure of the dividend.*

IV. *Divide, as before, until all the remaining figures of the dividend have been brought down and divided.*

V. *If any partial dividend will not yield an integral quotient, write a cipher in the quotient, bring down the next figure of the dividend, and proceed as before.*

VI. *Write the final remainder, if any, over the divisor, and annex it to the integral part of the quotient.*

Note. — If the product of the divisor by the quotient figure is larger than the partial dividend, the quotient figure is *too large;* on the other hand, if the remainder is larger than the divisor, the quotient figure is *too small.*

63. Examples in Multiplication and Division.

Perform the indicated operations upon the following numbers, and verify each by its converse operation:

1. 5 × 524.
2. 6 × 735.
3. 7 × 877.
4. 9 × 385.
5. 8 × 273.
6. 2 × 394.
7. 3 × 492.
8. 7 × 654.
9. 5 × 792.
10. 6 × 391.
11. 9 × 648.
12. 8 × 537.
13. 10 × 283.
14. 11 × 333.
15. 12 × 673.
16. one-half × 482.
17. one-third × 963.
18. one-fourth × 848.
19. one-fifth × 555.
20. one-sixth × 672.
21. one-third × 621.
22. one-fourth × 824.
23. one-fifth × 560.
24. one-sixth × 642.
25. one-eighth × 832.
26. one-ninth × 927.
27. one-tenth × 880.
28. one-twelfth × 400.
29. one-ninth × 175.
30. 8 × 2463.
31. 9 × 4735.
32. 5 × 8973.
33. 6 × 3579.
34. 12 × 6924.
35. 13 × 8431.
36. 14 × 6842.
37. 15 × 9364.
38. 16 × 6427.
39. 17 × 3105.
40. 18 × 2000.
41. 19 × 2040.
42. 20 × 6789.
43. 25 × 8642.
44. 36 × 1235.
45. 58 × 7935.
46. 89 × 8893.
47. 93 × 4269.
48. 75 × 3570.
49. 87 × 6789.
50. 99 × 5787.
51. 6781 × 39.
52. 8560 × 53.

53. 7680 × 29.
54. 9666 × 55.
55. 8073 × 67.
56. 8765 × 73.
57. 7895 × 97.
58. 3046 × 48.
59. 7008 × 61.
60. 8763 × 24.
61. 9273 × 86.
62. 8357 × 76.
63. 7382 × 47.
64. 6123 × 152.
65. 3246 × 295.
66. 4768 × 357.
67. 6443 × 462.
68. 8961 × 527.
69. 9344 × 871.
70. 3540 × 375.
71. 8973 × 865.
72. 7235 × 657.
73. 2335 × 892.
74. 4371 × 963.
75. 5575 × 551.
76. 3358 × 369.
77. 8117 × 649.
78. 9329 × 728.
79. 29375 × 2673.
80. 35593 × 5345.
81. 46738 × 6476.
82. 928376 × 8295.
83. 456973 × 6894.
84. 764289 × 7264.
85. 896728 × 6479.
86. 731645 × 9347.
87. 929695 × 8975.
88. 257343 × 74689.
89. 396647 × 86728.
90. 972869 × 79120.
91. 3745643 × 89691.
92. 7642531 × 67231.
93. 8963378 × 57243.
94. 9338726 × 30050.
95. 8936424 × 24489.
96. 4937258 × 39473.
97. 8312764 × 27621.
98. 9040657 × 30470.
99. 5313456 × 27359.
100. 6394336 × 11425.
101. 3947358 × 23456.
102. 9218347 × 53462.
103. 2645467 × 60735.
104. 7304856 × 73378.
105. 1819784 × 48235.
106. 492 ÷ 2.
107. 648 ÷ 4.
108. $\frac{726}{6}$.
109. $\frac{1}{3}$ of 963.
110. 843 ÷ 3.
111. 942 ÷ 5.
112. 764 ÷ 6.

113. 894 ÷ 8.
114. 973 ÷ 9.
115. 873 ÷ 3.
116. 950 ÷ 5.
117. 975 ÷ 7.
118. 864 ÷ 2.
119. 731 ÷ 1.
120. 342 ÷ 4.
121. 845 ÷ 7.
122. 633 ÷ 3.
123. 482 ÷ 8.
124. 697 ÷ 3.
125. 749 ÷ 4.
126. 328 ÷ 9.
127. 349 ÷ 3.
128. 376 ÷ 5.
129. 579 ÷ 6.
130. 756 ÷ 2.
131. 849 ÷ 6.
132. 497 ÷ 9.
133. 82472 ÷ 24.
134. 93684 ÷ 35.
135. 77666 ÷ 36.
136. 93472 ÷ 49.
137. 64379 ÷ 57.
138. 72893 ÷ 33.
139. 92173 ÷ 77.
140. 85967 ÷ 89.
141. 92573 ÷ 97.
142. 34254 ÷ 39.
143. 97531 ÷ 99.
144. 647342 ÷ 125.
145. 426801 ÷ 234.
146. 735761 ÷ 342.
147. 873456 ÷ 731.
148. 9736480 ÷ 941.
149. 872641396475 ÷ 46937.
150. 768123862469 ÷ 59735.
151. 459643731892 ÷ 693495.
152. 937864378104 ÷ 693725.
153. 786420000959 ÷ 763541.
154. 897612357431 ÷ 987643.
155. 318706403726 ÷ 896739.
156. 917630123897 ÷ 345797.
157. 216317621319 ÷ 943349.
158. 167358427659 ÷ 783441.
159. 296718317483 ÷ 863499.
160. 300450000339 ÷ 798567.

64. Examples for Review.

1. A gentleman owned U.S. bonds to the amount of $13275. At one time he disposed of $3000; at another, of $3525; at another, of as much as at both of the former times. What was the amount left?

2. If a man earns 90 dollars per month, and his expenses are 65 dollars per month, how long will it take him to pay for a house worth 1200 dollars?

3. From a cistern containing 25719 gallons of oil, 15387 gallons were drawn out; after which 17100 gallons were pumped in. How much oil was then in the cistern?

4. What is the quotient of 584379 divided by three times the difference between 475 and 218?

5. An estate valued at 25765 dollars was divided among four men. The first received 5278 dollars; the second, 7300 dollars; the third, 8105 dollars. How much did the fourth receive?

6. Mr. Ball had three stores, valued respectively at 21350 dollars, 9224 dollars, and 10421 dollars; also a farm, worth 7260 dollars. He gave to his daughter one-third of the value of the stores and one-fourth of the value of the farm, and divided the remainder equally between his two sons. How much did each receive?

7. If a man's income is $1600 a year, and his expenses $850, how many years will elapse before he will be worth $20000, if he is worth $8750 at the present time?

8. A mill, valued at 43458 dollars, was bought by three men. The first furnished one-sixth of the capital, the second twice as much as the first, and the third the remainder. How much did each man furnish?

9. Mr. Horne sells 135 bushels of potatoes at 2 dollars a bushel, 12 bales of cotton at 35 dollars a bale, and 3 tons of hay at 9 dollars a ton; he takes in payment a pair of mules worth 83 dollars each, a wagon worth 47 dollars, a buggy worth 125 dollars, and a set of harness worth 27 dollars. One-half of the remainder he spends for groceries, and deposits the balance in the bank. What is the amount of his deposit?

10. If one man's wages are equal to the wages of 5 boys, and 15 men earn 3375 dollars, how much does a boy earn?

11. If 25 men are 25 days putting up the walls of a factory, and each man lays 1784 bricks a day, how many bricks will there be in the walls of the building?

12. A cistern has two pipes, one emptying into it 78 gallons of water an hour, and the other drawing from it 56 gallons an hour. How many gallons of water will be in the cistern at the end of 37 hours?

13. The school funds of the town B—— are 6523 dollars, out of which the superintendent is paid 1600 dollars a year; the principal of the high school 1500 dollars; each of eight grammar school teachers 40 dollars a month, for 9 months; the janitor 12 dollars a month, for 9 months; and 75 dollars a year is expended for fuel and repairs. What increase of salary could each grammar school teacher receive if the surplus were divided among them?

14. If two men start from the same place, and travel in opposite directions, the one at the rate of 45 miles a day, and the other at the rate of 32 miles a day, how far apart will they be at the end of 17 days?

15. How far apart will they be if they travel in the same direction?

16. How many pounds of lard, at 18 cents a pound, will pay for 582 pounds of sugar, at 9 cents a pound?

17. If the driving wheel of a locomotive makes 4 revolutions per second, and at each revolution moves 18 feet, how far will it move in an hour, or 3600 seconds?

18. What is the amount of a man's bill who bought 12 yards of silk, at 75 cents a yard; 15 yards of flannel at 68 cents a yard; 18 yards of lace at 37 cents a yard?

19. A farmer sold 784 pounds of bacon at 15 cents a pound, and took, in payment, coffee at 20 cents a pound. How many pounds of coffee did he receive?

20. The earnings of a factory for a year were 375854 dollars, and the expenses were 213423 dollars. What were the profits?

21. In a certain boarding house there are 75 boarders at 25 dollars a month. The house rent is 117 dollars a month; table expenses, 750 dollars a month; servants' hire, 125 dollars a month; gas, water, fuel, etc., 150 dollars a month. What are the profits of the house for a year, or 12 months?

22. The sum of 51679680 dollars is to be distributed equally among a certain number of men, each to receive 84032 dollars. How many men are to receive the money?

23. Texas has an area of 280000 square miles, and Georgia's area is 52009 square miles. If Texas be divided into 4 states of equal size, how much larger than Georgia would each of them be?

24. A New York merchant sent his agent in Cincinnati 3529 dollars to be invested in bacon. After deducting his commission of 72 dollars, he expended the money in bacon at 20 cents (or $\frac{1}{5}$ of a dollar) a pound. How many pounds did he buy?

25. Mr. K. bought a piece of property, for which he paid 78000 dollars. He spent 3784 dollars on it for repairs, 780 dollars for taxes, and 390 dollars for insurance; after which he sold it for 85000 dollars. Did he gain or lose by the transaction, and what amount?

26. A drover bought 77 sheep of one farmer, 84 of another, 26 of another, and 38 of another. He sold one-third of them at 2 dollars a head, and the remainder at 3 dollars a head. What did he receive for the entire lot?

27. The dividend is 468002659, and the divisor 9497. What is the remainder?

28. What is that number the factors of which are 384, 97, 712, and 278?

29. 645600445 multiplied by 77386 divided by 931605 gives what quotient?

30. The yield of an oil mill for one month is 30186 gallons. What is its yield for one year, or 12 months? How many casks, each holding 43 gallons, will be required to hold it?

31. Mr. Brooks left an estate, valued at 8500 dollars, to be divided equally among 3 sons and 2 daughters. What did each receive?

32. A man is 30 years old; $\frac{1}{2}$ of his age is $\frac{1}{4}$ of his father's age. How old is his father?

33. A man bought 3 loads of hay; the first weighed 6365 pounds, the second weighed 5984 pounds, and the third weighed 3596 pounds. How many pounds did he buy?

34. A man exchanged a piece of property worth 18750 dollars, and a piece worth 64875 dollars, for rolling-mill stock valued at 310504 dollars, paying the balance in money. How much money did he pay?

35. A man on going into business had 17948 dollars. The first year he gained 982 dollars, the second year 1125 dollars, and the third year he lost 2107 dollars. How much was he then worth?

36. If the remainder is 325, the quotient 467, and the divisor 43 more than the sum of both, what is the dividend?

37. A farmer sold 185 barrels of apples at 4 dollars a barrel, and with the money thus obtained bought 8 horses, at 78 dollars apiece, and his year's supply of groceries. How much did his groceries cost him?

38. Six men bought an interest in a mine for 36420 dollars. The first paid 12140 dollars; the second, 3035 dollars less than the first; the third, 346 dollars; the fourth, 6070 dollars more than the third; the fifth, 1821 dollars less than the fourth. How much did the sixth pay?

39. What length of time elapsed between the birth of George Washington in 1732 and the birth of Robert E. Lee in 1807?

40. A's liabilities are 450120 dollars. His assets are as follows: railroad stock, 350000 dollars; mining stock, 225100 dollars; his home, 4000 dollars; and a store of goods, valued at 96000 dollars. How much is he worth?

41. A gentleman whose annual income is 7500 dollars, spends 106 dollars a week. There are 52 weeks in a year. Does he save, or run in debt, and how much?

42. Two persons, A and B, are indebted to C. A owes 3712 dollars, which is the smaller debt, and the difference between the debts is 173 dollars. What is the amount of their indebtedness?

43. Suppose the driving wheel of a locomotive be 22 feet in circumference, and to make 272 revolutions in a minute; what distance will the engine move in 1 hour, or 60 minutes, there being 5280 feet in a mile?

44. A father left an estate worth 65000 dollars to be divided among his children. The eldest son was to receive 17400 dollars; his only daughter 1400 less than her oldest brother; the remainder was to be divided equally among his four other sons. What amount did they each receive?

45. How many prisoners can be maintained on 25972 dollars per annum, if each prisoner requires, on an average, 86 dollars per annum?

46. A merchant bought 182 yards of cloth at 3 dollars a yard, and 212 yards at 5 dollars a yard. He sold it all at 4 dollars a yard. How much did he lose?

47. The salary of the President of the United States is 50000 dollars a year. How much can he spend daily and save 4375 dollars annually?

48. The annual income of a certain city is 4364900 dollars, and its expenses 3298700 dollars. How much will that city save in 7 years?

49. The diameter of the sun is 786144 miles, and that of the earth is 7912 miles. What is the difference in their diameters?

50. A merchant bought 115 yards of velvet for 690 dollars, and sold 58 yards of it at cost. What did the velvet sold amount to?

51. A bank failed, being in debt to the amount of 584325 dollars. On winding up its affairs, it was found to own a building valued at 57650 dollars, office furniture worth 15738 dollars, notes on other banks to the amount of 60503 dollars, and 7433 dollars in gold. By how much did its indebtedness exceed its assets?

CONTRACTED PROCESSES.

65. To multiply by 10, 100, 1000, etc.; that is, by any order of unit higher than the first.

Annexing a zero to a number removes each figure one place to the left; that is, multiplies its value, and therefore that of the number, by 10; annexing two zeros removes each figure two places to the left, and therefore multiplies the number by 100; annexing three zeros multiplies by 1000; four, by 10000; etc.

1. Multiply 25 by 10.

2. Multiply 176 by 10, by 100, by 1000.

3. Multiply 359 by 20, by 300, by 5000.

SUGGESTION. — $20 = 10 \times 2$; that is, twice 10 times a number is 20 times the number; therefore, to multiply by 20, multiply the number by 10 and the product by 2.

4. Multiply 793 by 50, by 600, by 700.

5. Multiply 3251 by 30, by 800, by 9000.

66. To divide by 10, 100, 1000, etc.; that is, by any order higher than the first.

Rejecting a figure to the right of a number removes each figure one place to the right; that is, divides its value, and therefore that of the number, by 10.

On the same principle, rejecting two figures divides by 100; three figures by 1000; etc.

1. Divide 350 by 10. *Ans.* 35.

2. Divide 347 by 10. *Ans.* $34\frac{7}{10}$.

3. Divide 6892 by 200.

PROCESS.

$$2|00\overline{)68|92}$$
$$34\tfrac{92}{200}$$

EXPLANATION. — $200 = 2 \times 100$; $\frac{1}{200} = \frac{1}{2}$ of $\frac{1}{100}$. Reject two figures and divide the remaining figures by 2, placing the rejected figures as a remainder over the divisor (200).

4. Divide 62500 by 2500, by 5000.

5. Divide 172800 × 14400 by 120.

6. Divide 6729 by 300.

67. To multiply by aliquot parts of a hundred.

25 is $\frac{1}{4}$ of 100; $33\frac{1}{3} = \frac{1}{3}$ of 100; $12\frac{1}{2} = \frac{1}{8}$ of 100; therefore, to multiply by 25, multiply by 100 and divide the product by 4.

1. Multiply 617 by 25.

PROCESS.

$$617 \times 25 = 61700 \div 4 = 15425.$$

CHAPTER IV.

MULTIPLES AND MEASURES.

Properties and Relations of Integers.

68. A **Measure** of any given number is an exact integral divisor of it. Thus 2 is a measure of 8; for 2 is an integer, and divides 8 exactly; that is, without a remainder.

69. A **Composite Number** is one that can be measured in terms of two or more numbers. It is a product of two or more factors. Thus 6 is a composite number, because it can be measured by 2 or 3, its factors.

70. A **Prime Number** is one that cannot be measured by any number other than itself or 1. Thus 3, 5, 7, 11 are prime numbers.

71. A **Prime Measure** of a number is a prime number that will exactly divide it.

72. A **Composite Measure** of a number is a composite number that will exactly divide it.

Analysis of Composite Numbers.

73. **1.** What are the prime measures of 105?

PROCESS.

$$\begin{array}{r|r} 3 & 105 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

Explanation. — By dividing, it is found that 105 is composed of two factors: 3 and 35; 3 is prime, 35 composite. By dividing again, it is found that the composite measure, 35, is composed of two prime measures: 5 and 7. Therefore, the prime measures of 105 are found to be 3, 5, and 7. That is, $105 = 3 \times 5 \times 7$.

74. RULE.—*Divide the given number by the least number that will exactly divide it, and each quotient in the same way, until a quotient is obtained that is a prime number. The divisors and the last quotient will be the prime measures sought.*

75. Find the prime measures of each of the following:

1. 32.
2. 60.
3. 72.
4. 55.
5. 272.
6. 360.
7. 520.
8. 972.
9. 121.
10. 144.
11. 876.
12. 1000.
13. 1575.
14. 8175.
15. 9333.
16. 9999.
17. 42852.
18. 21896.
19. 35800.
20. 96472.
21. 87581.
22. 25375.
23. 45965.
24. 85864.

THE GREATEST COMMON MEASURE.

76. A **Common Measure** of two or more numbers is a number that will exactly divide, or measure, each of them. Thus 2 is a common measure of 4, 6, 8.

77. The **Greatest Common Measure** of two or more numbers is the greatest number that will exactly divide each of them. Thus 6 is the greatest common measure of 12 and 18.

NOTE.—For the sake of brevity the Greatest Common Measure may be denoted by the letters G. C. M.

78. A measure is known indifferently as a divisor or as a factor. The greatest common measure is also called the greatest common divisor, or the highest common factor.

79. 1. Find the common prime measures of 45 and 60, and their G. C. M.

PROCESS.

$$\begin{array}{r|l} 3 & 45,\ 60 \\ \hline 5 & 15,\ 20 \\ \hline & \ \ 3,\ \ 4 \end{array}$$

EXPLANATION.—By trial it is found that 3, which is prime, will exactly divide both numbers; 3 is therefore a common prime measure of the numbers. Again, 5, which is also prime, is found exactly to divide both quotients; 5 is therefore a common prime measure of the numbers. The last quotients have no common divisor; that is, they are prime with respect to each other.

Whence, 3 and 5 are the only common prime measures of 45 and 60; but each number is divisible by the product of its prime measures. Hence the common measures of 45 and 60 are 3, 5, and 3 × 5, or 15; of which 15 is the greatest. 15 is therefore the G.C.M. of 45 and 60.

80. Rule. — *Divide the given numbers by any prime number that will exactly divide all of them, and continue to divide the several sets of quotients in the same way until they have no common divisor. The divisors will be the common prime measures, and their product the G. C. M.*

81. 1. Find the G. C. M. of 21 and 81.

PROCESS.

```
21)81(3
   63
   --
   18)21(1
      18
      --
       3)18(6
         18
         --
```

Explanation. — The G. C. M. of two numbers cannot be greater than the less number. If, then, the less number exactly divides the greater, the less is the G. C. M. By trial, 81 divided by 21 gives a quotient 3 and a remainder 18. The division is not exact, and 21 is not the G.C.M. If the remainder, 18, exactly divides 21, it will also divide 81, which is equal to 3 times 21 plus 18. By trial, 21 divided by 18 gives a quotient 1 and a remainder 3. The division is not exact, and 18 is not the G.C.M. If the remainder, 3, exactly divides 18, it will also divide 21, which is 18 plus 3. If it divides 18 and 21, it will also divide 81, which is 3 times 21 plus 18. By trial, 18 divided by 3 gives a quotient 6 without a remainder. The division is exact, and 3 is the G.C.M. of 21 and 81.

82. Rule. — *Divide the greater number by the less, and, if the division is not exact, the first divisor by the first remainder, the second divisor by the second remainder, and so on, until an exact divisor is obtained. The last divisor will be the G. C. M.*

Note. — If more than two numbers are given, find the G.C.M. of any two of them, then of this G. C. M. and another of the numbers, and so on. The last divisor will be the G.C. M.

Note to Teacher. — This last method of finding the G. C. M. has some practical superiority over the first method; but its explanation is so abstract that few pupils below the high school grades are able to understand the rationale of its process.

The method is borrowed from geometry, where it is used to find the G.C.M. of two or more lines. If the teacher prefers this to the first and simpler method, he might concrete it by finding the G. C. M. of two lines, *A* and *B*. But it is the experience of the author that the average grammar school pupil is injured rather than benefited by an effort to master it; and it is suggested that the second, or geometrical method, be omitted altogether, or reserved for the general review.

83. Find the prime measures of the following; and then their G. C. M.'s.

1. 4 and 12.
2. 10 and 15.
3. 12 and 18.
4. 24 and 32.
5. 36 and 48.
6. 60 and 108.
7. 72 and 81.
8. 121 and 132.
9. 56 and 63.
10. 63 and 105.
11. 246 and 342.
12. 247 and 323.
13. 272 and 425.
14. 330 and 720.
15. 825 and 2700.
16. 630 and 936.
17. 324 and 612.
18. 5184 and 6912.
19. 3444 and 3556.
20. 2472 and 3264.
21. 5184 and 6914.
22. 12, 18, 64, and 96.
23. 24, 36, 72, and 81.

CANCELLATION.

84. Cancellation is a method of shortening division by rejecting, or cancelling, all equal factors that occur in both dividend and divisor.

1. Divide 36 by 12.

PROCESS.

$$\frac{36}{12} \times \frac{\cancel{3} \times \cancel{2} \times 3 \times \cancel{2}}{\cancel{3} \times \cancel{2} \times \cancel{2}} = 3.$$

Rejecting a factor common to both divisor and dividend does not alter the quotient, becauses it diminishes both in the same ratio.

2. Divide 42 by 12.
3. Divide 60 by 18.
4. Divide 372 by 24.
5. Divide 81 by 27.
6. Divide 28 × 16 × 48 by 21 × 8 × 7.

THE LEAST COMMON MULTIPLE.

85. A **Multiple** of a given number is a number that is exactly divisible by the given number.

86. A **Common Multiple** of two or more numbers is a number that is exactly divisible by each of the given numbers. Thus 24 is a common multiple of 2, 3, 4, and 6.

NOTE.—The Least Common Multiple may be denoted by the letters L. C. M.

87. The **Least Common Multiple** of two or more numbers is the least number that is exactly divisible by each of the given numbers. Thus 12 is the L. C. M. of 2, 3, 4, and 6.

It is evident that a common multiple of two numbers may be their product, or any integral number of times their product; that is, the multiple may be composed of all the factors of both numbers, and any number of factors besides. Thus 24, 48, 72 are multiples of 6 and 4.

It is equally evident that the L. C. M. of two numbers must be composed of all the factors of each, without repeating common factors; that the L. C. M. of 6 and 4 must be 6 multiplied by the factor of 4 not found in 6, viz. 2; which gives $2 \times 6 = 12$ for the L. C. M.; that the L. C. M of two numbers is their product divided by their G. C. M. Thus the L. C. M. of 6 and 4 is $\frac{6 \times 4}{2} = 12$.

The L. C. M. of three or more numbers must be composed of all the factors of each number without repeating any factor common to two or more of the numbers. The L. C. M. of 9, 6, and 4 must be composed of the factors of 9 and 4, which is 36. One of the factors of 6 is found in 9 and the other in 4, and these factors are not repeated in the L. C. M.

88. 1. Find the L. C. M. of 9, 6, and 4.

PROCESS.

$$\begin{array}{r|l} 3 & 9,\ 6,\ 4 \\ \hline 2 & 3,\ 2,\ 4 \\ \hline & 3,\ 1,\ 2 \end{array}$$

$$\text{L. C. M.} = 3 \times 2 \times 3 \times 2 = 36.$$

Explanation. — Any prime factor that exactly divides at least two of the numbers is a common factor of those two numbers, and is to be used but once as a factor of the L. C. M.

By inspection it may be seen that 3 and 2 are the only prime factors common each to two of the numbers 9, 6, and 4. Divide by 3, bringing down the undivided number on a line with the quotients; then divide by 2; the last quotients are prime to each other, and the division cannot be continued.

The L. C. M. is the continued product of the divisors 3, 2 by the quotients 3, 2, or $3 \times 2 \times 3 \times 2 = 36$.

89. Rule. — *Write the numbers on a horizontal line, and divide by the least prime factor that will exactly divide two or more of them, writing in a line below the quotients and the undivided numbers. Continue to divide as before until there is no prime number greater than* 1 *that will exactly divide two or more of them.*

The continued product of the divisors and the numbers in the last line will be the L. C. M. sought.

90. Find the L. C. M. of the following:

1. 16 and 28.
2. 37 and 25.
3. 5, 9, and 15.
4. 2, 4, and 8.
5. 9, 15, and 24.
6. 8, 40, and 120.
7. 20, 30, and 50.
8. 8, 12, 16, and 18.
9. 5, 6, 9, and 10.
10. 3, 4, 6, 8, 24, and 40.
11. 9, 11, 18, 22, and 36.
12. 9, 27, 81, 243, and 729.
13. 60, 90, 25, 105, and 9.
14. 12, 54, 28, 63, and 21.
15. 35, 28, 27, 24, and 21.
16. 96, 72, 56, 32, and 28.
17. 56, 36, 32, 27, and 24.
18. 28, 32, 21, 18, and 14.
19. 468 and 923.
20. 625, 735, 860, and 915.

CHAPTER V.

DENOMINATE NUMBERS.

91. **Quantity** is the amount or extent of anything; and a unit is that standard established by law, or custom, in terms of which a given kind of quantity is measured.

92. **Measuring** is the process of finding how many times the unit, the given quantity is; and thus gives rise to number.

93. Quantity answers the question "how much"; number, the question "how many."

94. A quantity is frequently measured in terms of several related orders of units, called denominations, and derived by taking multiples and subdivisions of the standard unit; as, from the standard yard there are derived by multiplication the superior denominations, rods and miles; and by subdivision, the inferior denominations, feet and inches.

95. A number expressed in one denomination is called a **Denominate Number.**

96. A number expressed in two or more denominations is called a **Compound Denominate Number.**

97. The *rate* by which the denominations increase and decrease is termed a **Scale.** The scale of the Arabic notation is 10; those of the denominate numbers are for the most part irregular or variable. Thus 12 inches make a foot; 3 feet, a yard; 5½ yards, a rod; and 320 rods, a mile.

98. There may be quantity, and therefore number, with respect to:

(1) Extension, which is the property of being extended in space.

(2) Capacity, which is the property of containing, or holding.

(3) Weight, which is the measure of the force of gravity.

(4) Time, which is the measure of duration.

(5) Angularity, which is the measure of the difference of direction.

(6) Value, which is expressed in terms of money.

EXTENSION.

99. Extension has three dimensions: length, breadth, and thickness.

100. A **Line** has length and direction. It is mere distance.

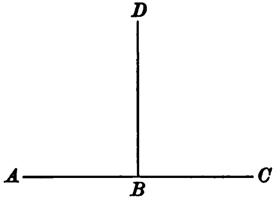
101. A **Straight Line** is a line that does not change its direction throughout its extent, — it is the shortest distance between two points.

102. A **Surface,** or plane, has length and breadth.

103. A **Solid** has length and breadth and thickness.

104. An **Angle** is the measure of divergence from a given direction. To illustrate: A soldier is facing the north; at the command "right face," he wheels to the east, making a fourth of an entire revolution. This difference in the directions of his first and second facings is called an angle. In this case it is a fourth of a revolution, or a right angle, equal to 90 degrees. If he "right face" again, he will face the south, the opposite of his first facing, making a half revolution, or two right angles, called a straight angle. Another "right face" completes three-fourths of a revolution, or three right angles. Yet another "right face" brings the soldier to his original facing, the north, and completes an entire revolution, or four right angles.

105. Angles are represented by the divergence of lines proceeding from a common point called the vertex. Thus, *A B D* and *D B C* are angles, and *B* their common vertex.



106. When one straight line meets another straight line, making the two adjacent angles equal, each angle is a right angle, and their lines are said to be perpendicular to each other.

NOTE. — The unit of linear, or long, measure is a *straight* line, one yard. Therefore, only straight lines can be directly measured; crooked or curved lines, to be measured, must first be reduced to straight lines.

107. **Parallel Lines** are lines that have the same direction.

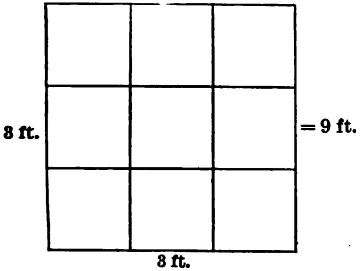
108. A **Square** is a surface bounded by four equal sides, and having four right angles.

109. The standard unit of surface measurement is the square yard. Since the square is a right-angled or rectangular figure, and can be applied only to rectangular figures, other plane figures cannot be directly measured; triangles, circles, etc., to be measured, must first be reduced to rectangles.

110. A **Cube** is a solid bounded by six equal squares. It is therefore rectangular.

111. The standard unit of solid measurement is a cubic yard. Since the cubic yard is rectangular, and can be applied only to rectangular solids, other solid figures cannot be directly measured; cones, cylinders, and spheres must, to be measured, be reduced to rectangular figures.

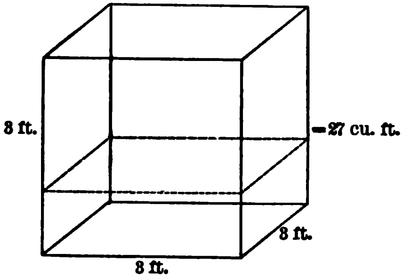
112. The area of a rectangular plane is equal to the product of its length by its breadth expressed in units of the same denomination.



The lines divide the square yard into 3 rows, with 3 sq. ft. in each row. Hence the square yard contains 3 times 3 sq. ft. = 9 sq. ft. Notice that the multiplier is abstract; that feet are not multiplied by feet; that the expression feet × feet does not denote multiplication, but simply indicates the dimensions of the plane.

113. The volume of a rectangular solid is equal to the product of its length, breadth, and thickness.

Let it be required to find the solidity of a cubical box whose length, breadth, and thickness is each 3 ft.



The floor of the box is a square 3 ft. × 3 ft.; it will therefore contain 9 sq. ft. If sand be poured into the box until the floor is covered to the depth of 1 ft., there will be 9 cu. ft. of sand in the box. If sand be poured another foot deep, there will be another layer of sand containing 9 cu. ft. Another like pouring will fill the box, when it will contain 3 layers of sand, each layer containing 9 cu. ft.; whence the box will contain 3 times 9 cu. ft., or 27 cu. ft. = 3 ft. × 3 ft. × 3 ft.

114. LINEAR OR LONG MEASURE.

12 Inches (in.)	make a Foot (ft.).
3 Feet	make a Yard (yd.).
5½ Yards	make a Rod (rd.).
320 Rods	make a Mile (mi.).

In geographical and astronomical calculations.

1.15 Statute Miles	make a Geographical Mile.	
3 Geo. Miles	make a League.	
60 Geo. Miles or 69.16 Stat. Miles	make a Degree	of Latitude on a Meridian, or of Longitude on the Equator.
360 Degrees	make the Circumference of the Earth.	

115. Square Measure.

144 Square Inches	make a Square Foot (sq. ft.).
9 Square Feet	make a Square Yard (sq. yd.).
30¼ Square Yards	make a Square Rod (sq. rd.).
160 Square Rods	make an Acre (A.).
640 Acres	make a Square Mile (sq. mi.).

116. Solid Measure.

1728 Cubic Inches	make a Cubic Foot (cu. ft.).
27 Cubic Feet	make a Cubic Yard (cu. yd.).
128 Cubic Feet	make a Cord(cd.).

117. Lumber Measure.

A lumber foot is one foot long, one foot broad, and one inch thick; hence a cubic foot is equal to 12 lumber feet.

The lumber feet in a given quantity of lumber may be found by multiplying the length in feet by the breadth in feet by the thickness in inches; or by multiplying the cubical contents in feet by 12.

Lumber is bought and sold by the 1000 ft.

118. Measures of Capacity.

Liquid Measure.

4 Gills (gi.)	make a Pint (pt.).
2 Pints	make a Quart (qt.).
4 Quarts	make a Gallon (gal.).

The standard unit of Liquid Measure is the gallon, containing 231 cu. in. In finding the capacity of reservoirs, 31½ gallons is considered a barrel (bbl.) and 63 gallons a hogshead (hhd.). In commerce the barrel and hogshead are not fixed measures.

119. *Dry Measure.*

2 Pints make a Quart (qt.).
8 Quarts make a Peck (pk.).
4 Pecks make a Bushel (bu.).

The standard unit of Dry Measure is the Winchester bushel, containing 2150.42 cu. in.

The denominations, pint, quart, gallon of dry measure, are greater than the like denominations, liquid measure; the dry quart being about 67⅕ cu. in., and the liquid quart only 57¾ cu. in.

120. *Apothecaries' Liquid Measure.*

This measure is used in compounding liquid medicine.

60 Minims (♏) make a Fluid Drachm (*f*ʒ).
8 Fluid Drachms make a Fluid Ounce (*f*℥).
16 Fluid Ounces make a Pint (O.).
8 Pints make a Gallon (Cong.).

121. MEASURES OF WEIGHT.

Avoirdupois Weight.

16 Ounces (oz.) make a Pound (lb.).
25 Pounds make a Quarter (qr.).
100 Pounds make a Hundredweight (cwt.).
20 Hundredweight make a Ton (T.).

Avoirdupois Weight is used in measuring all the coarser articles of commerce; as grain, groceries, coal, etc. The standard unit of measure is the pound, containing 7000 grains.

122. *Apothecaries' Weight.*

20 Grains	make a Scruple (℈).
3 Scruples	make a Dram (ʒ).
8 Drams	make an Ounce (℥).
12 Ounces	make a Pound (℔).

Apothecaries' Weight is used by druggists in preparing dry prescriptions. The pound contains 5760 grains. Medicines in quantity are bought and sold by Avoirdupois Weight.

123. *Troy Weight.*

24 Grains	make a Pennyweight (dwt.).
20 Pennyweights	make an Ounce (oz.).
12 Ounces	make a Pound (lb.).

Troy Weight is used in weighing gold, silver, jewels, precious stones, etc.

124. MEASURES OF TIME.

60 Seconds (sec.)	make a Minute (min.).
60 Minutes	make an Hour (hr.).
24 Hours	make a Day (da.).
7 Days	make a Week (wk.).
365 Days	make a Year (yr.).
366 Days	make a Leap Year (yr.).
100 Years	make a Century (cen.).
1000 Years	make a Millenium.

In ordinary business transactions 30 days is accounted a month.

The number of days in each of the calendar months may be readily remembered by means of the following lines:

"Thirty days hath September,
April, June, and November.
All the rest have thirty-one,
Except the second month alone,
Which has but twenty-eight in fine
'Till leap-year gives it twenty-nine."

125. Angular Measure.

60 Seconds (″) make a Minute (′).
60 Minutes make a Degree (°).
30 Degrees make a Sign (S.).
360 Degrees make a Circumference (Cir.).

The circumference of a circle is an *angular* as well as a linear magnitude. If one should follow the circumference of a circle completely around to the point whence he started the circuit, he would make as complete a revolution of the direction of his facing as though he had made four "right faces." That is, one-fourth of the circuit is equal in change of direction to one "right face," one-half circuit to two "right faces," etc. Whence a circumference is equal to four right angles, or 360 degrees; a semicircumference to two right angles, or 180 degrees; a quadrant to one right angle, or 90 degrees.

Measures of Value.

126. *Federal Money.*

10 Mills (m.) make a Cent (ct.).
10 Cents make a Dime (d.).
10 Dimes make a Dollar ($).
10 Dollars make an Eagle (E.).

127. *English Money.*

4 Farthings (far.) make a Penny (*d.*).
12 Pence make a Shilling (*s.*).
20 Shillings make a Pound (£).

128. *French Money.*

10 Millimes make a Centime (ct.).
10 Centimes make a Decime (dc.).
10 Decimes make a Franc (fr.).

The English pound is equal to $ 4.86 + U. S. money, and the French franc to $ .193.

129. Miscellaneous Tables.

12 Things make a Dozen.
12 Dozen make a Gross.
12 Gross make a Great Gross.
20 Units make a Score.

130. *Paper.*

24 Sheets make a Quire.
20 Quires make a Ream.
2 Reams make a Bundle.
5 Bundles make a Bale.

Metric Tables.

131. The **Meter** is the unit of length and is (nearly) the one ten-millionth part of the distance on the earth's surface from the equator to either pole, or 39.37 + inches.

10 Millimeters (mm.) make a Centimeter (cm.).
10 Centimeters make a Decimeter (dm.).
10 Decimeters make a Meter (M.).
10 Meters make a Dekameter (Dm.).
10 Dekameters make a Hektometer (Hm.).
10 Hectometers make a Kilometer (Km.).
10 Kilometers make a Myriameter (Mm.).

132. The **Are** is the unit of land measure, and is a square whose side is a dekameter.

100 Centiares (ca.) make an Are (A.).
100 Ares make a Hektare (Ha.).

133. The **Liter** is the unit of capacity, and is equal to 1.05 + qt. liquid measure.

10 Milliliters (ml.) make a Centiliter (cl.).
10 Centiliters make a Deciliter (dl.).
10 Deciliters make a Liter (L.).
10 Liters make a Dekaliter (Dl.).
10 Dekaliters make a Hektoliter (Hl.).
10 Hektoliters make a Kiloliter (Kl.), or stere.
10 Kiloliters make a Myrialiter (Ml.).

134. The **Gram** is the unit of weight, and is equal to 15.432 Troy grains.

10 Milligrams (mg.) make a Centigram (cg.).
10 Centigrams make a Decigram (dg.).
10 Decigrams make a Gram (G.).
10 Grams make a Dekagram (Dg.).
10 Dekagrams make a Hektogram (Hg.).
10 Hektograms make a Kilogram (Kg.) or (Kils.).
10 Kilograms make a Myriagram (Mg.).
10 Myriagrams make a Quintal.
10 Quintals make a Ton.

CHAPTER VI.

REDUCTION.

135. **Reduction** is the method of changing the *unit*, without altering the value, of an expression.

If the unit of an expression be changed to a less unit, the number of these units will be proportionally greater.

If the unit of expression be changed to a greater unit, the number of these units will be proportionally less.

Since a yard is three times as great as a foot, there will be one-third as many yards as feet, and three times as many feet as yards, in any given linear expression.

136. Changing the unit of an expression to a higher unit (or denomination) is called **Reduction Ascending.**

Again, changing the unit of an expression to a lower unit (or denomination) is called **Reduction Descending.**

Thus 36 inches = 3 feet, and 12 pints = 6 quarts, and 16 pecks = 4 bushels, are examples of *reduction ascending.*

Again, 1 yard = 36 inches, and 1 rod = $16\frac{1}{2}$ feet, and 1 gallon = 8 pints, are examples of *reduction descending.*

137. Reduction ascending and reduction descending are therefore *converse* processes, and either may be used to verify the other.

138. To express a number of a higher denomination in terms of a unit of a lower denomination.

1. Find how many gills there are in 5 gal. 3 qt. 1 pt. 2 gi.

PROCESS.

$$\begin{array}{l} \text{gal. qt. pt. gi.}\\ \;5\quad 3\quad 1\quad 2\\ \;\underline{4}\\ 20\text{ qt.}\\ \;\underline{3}\\ 23\text{ qt.}\\ \;\underline{2}\\ 46\text{ pt.}\\ \;\underline{1}\\ 47\text{ pt.}\\ \;\underline{4}\\ 188\text{ gi.}\\ \;\underline{2}\\ 190\text{ gi.} \end{array}$$

Explanation. — Since there are 4 times as many quarts as gallons in any expression, there are in 5 gal. 4 times 5 qt. = 20 qt.; 20 qt. and 3 qt. equal 23 qt.; 23 qt. equal twice as many pints, or 46 pt.; 46 pt. and 1 pt. = 47 pt.; 47 pt. equal 4 times as many gills or 188 gi.; 188 gi. and 2 gi. = 190 gi. Hence 5 gal. 3 qt. 1 pt. 2 gi. = 190 gi.

139. To express a number of a lower denomination in terms of a unit of a higher denomination.

1. Reduce 190 gills to higher denomination.

PROCESS.

$$\begin{array}{r|l} 4 & 190\text{ gi.}\\ \hline 2 & \;\;47\text{ pt. } 2\text{ gi.}\\ \hline 4 & \;\;23\text{ qt. } 1\text{ pt.}\\ \hline & \;\;\;\;5\text{ gal. } 3\text{ qt.} \end{array}$$

Explanation. — Since there are one-fourth as many pints as gills in any expression, there are in 190 gi. one-fourth of 190 pt. = 47 pt. and a remainder of 2 gi.; in 47 pt. there are one-half of 47 qt. = 23 qt. and 1 pt. remaining; in 23 qt. there are one-fourth of 23 gal. = 5 gal. and 3 qt. remaining. Hence in 190 gi., there are 5 gal. 3 qt. 1 pt. 2 gi.; and the last process verifies the first.

140. Reduce the following, and verify each process by its converse:

1. 21 days 12 hours 30 minutes 50 seconds, to seconds.
2. 5 tons 9 cwt. 2 qr. 15 lb. to pounds.
3. £12 16*s*. 9*d*. 3 far. to farthings.
4. 18965 far. to higher denominations.
5. 8972 oz. (avoirdupois) to higher denominations.

6. 16 rd. 4 yd. 2 ft. 9 in. to inches.

7. 257285 sec. to higher denominations.

8. 35672 gi. to higher denominations.

9. 47 lb. 9 oz. 13 dwt. 17 gr. to grains.

10. 3945678 gr. to higher denominations.

11. 1 mi. to inches.

12. 63360 in. to higher denominations.

13. 1 cu. yd. to cubic inches.

14. 46656 cu. in. to cubic yards.

Table of Equivalents for Linear Measure.

$$1 \text{ mile} = \begin{cases} 63360 \text{ in.} \\ 5280 \text{ ft.} \\ 1760 \text{ yd.} \\ 320 \text{ rd.} \end{cases}$$

15. Make a like table of equivalents for each of the tables of measures.

141. Addition and Subtraction.

1. Find the sum of 5 gal. 3 qt. 1 pt. 3 gi. and 7 gal. 2 qt. 1 pt. 2 gi.

PROCESS.

gal.	qt.	pt.	gi.
5	3	1	3
7	2	1	2
13	2	1	1

Explanation. — Write the numbers so that denominations of the same order shall stand in the same column, and proceed thus: 2, 5; 5 gi. = 1 pt. and 1 gi.; write the 1 gi. under the column of gills and "carry" the 1 pt. to the column of pints: 1, 2, 3; 3 pt. = 1 qt. and 1 pt.; write the 1 pt. under the column of pints and "carry" the 1 qt. to the column of quarts: 1, 3, 6; 6 qt. = 1 gal. and 2 qt.; write the 2 qt. under the column of quarts and "carry" the 1 gal. to the column of gallons: 2, 9, 13; write the 13 gal. under the column of gallons. Hence the sum of 5 gal. 3 qt. 1 pt. 3 gi. and 7 gal. 2 qt. 1 pt. 2 gi. is 13 gal. 2 qt. 1 pt. 1 gi.

RULE. — *Write the numbers so that denominations of the same order shall stand in the same column. Begin at the lowest denomination, and add as in abstract numbers, "carrying" one for as many units of the denomination added as it takes to make one of the next higher.*

Prove as in the addition of abstract numbers.

2. From 13 gal. 2 qt. 1 pt. 1 gi., take 5 gal. 3 qt. 1 pt. 3 gi.

PROCESS.

gal.	qt.	pt.	gi.
13	2	1	1
5	3	1	3
7	2	1	2

EXPLANATION. — Write the subtrahend under the minuend so that denominations of the same order shall stand in the same column. As 3 gi. cannot be taken from 1 gi., "borrow" 1 pt. from the next higher denomination; 1 pt. is equal to 4 gi., which, added to 1 gi., makes 5 gi.; 3 gi. taken from 5 gi. leaves 2 gi.; write the 2 gi. under the column of gills; as 1 pt. cannot be taken from 0 pt., "borrow" 1 qt. from the 2 qt.; 1 qt. is equal to 2 pt., which, added to the 0 pt., makes 2 pt.; 1 pt. taken from 2 pt. leaves 1 pt.; write the 1 pt. under the column of pints; as 3 qt. cannot be taken from 1 qt., "borrow" 1 gal. from the 13 gal.; 1 gal. is equal to 4 qt., which, added to the 1 qt., makes 5 qt.; 3 qt. taken from 5 qt. leaves 2 qt.; write the 2 qt. under the column of quarts; 5 gal. taken from 12 gal. leaves 7 gal.; write the 7 gal. under the column of gallons. Hence 5 gal. 3 qt. 1 pt. 3 gi. taken from 13 gal. 2 qt. 1 pt. 1 gi. leaves 7 gal. 2 qt. 1 pt. 2 gi.

RULE. — *Write the subtrahend under the minuend so that denominations of the same order shall stand in the same column.*

Begin with the lowest, and subtract each denomination of the subtrahend from the corresponding denomination of the minuend, as in abstract numbers.

If any denomination of the minuend is less than the corresponding denomination of the subtrahend, add to it as many units as it takes to make one of the next higher denomination, and subtract; regard the next higher denomination of the minuend as one less.

And proceed in the same way with each denomination.

Prove as in the subtraction of abstract numbers.

3. What is the sum of 35 bu. 2 pk. 5 qt. 1 pt., 27 bu. 0 pk. 5 qt. 1 pt., 23 bu. 3 pk. 6 qt. 1 pt., 19 bu. 3 pk. 7 qt. 1 pt., 52 bu. 1 pk. 4 qt. 0 pt., 29 bu. 3 pk. 2 qt. 1 pt.?

4. Find the sum of 25 da. 14 hr. 30 min. 45 sec., 53 da. 21 hr. 50 min. 35 sec., 12 da. 23 hr. 58 min. 57 sec., 35 da. 17 hr. 49 min. 37 sec., and 36 da. 12 hr. 15 min. 9 sec.

5. Find the sum of 25 gal. 2 qt. 1 pt. 3 gi., 18 gal. 3 qt. 1 pt. 2 gi., 27 gal. 3 qt. 1 pt. 1 gi., 34 gal. 2 qt. 0 pt. 3 gi., 19 gal. 1 qt. 1 pt. 3 gi., 36 gal. 3 qt. 1 pt. 2 gi.

6. What is the sum of £35 8*s.* 9*d.* 3 far., £43 13*s.* 5*d.* 3 far., £27 9*s.* 10*d.* 2 far., £18 17*s.* 11*d.* 2 far., £20 18*s.* 7*d.* 1 far.?

7. Find the sum of 25 lb. 9 oz. 13 pwt. 20 gr., 18 lb. 8 oz. 15 pwt. 17 gr., 23 lb. 11 oz. 18 pwt. 5 gr., 5 lb. 7 oz. 19 pwt. 16 gr., 10 lb. 10 oz. 10 pwt. 10 gr.

8. Find the difference between 25 lb. 50 oz. 8 pwt. 99 gr. and 18 lb. 8 oz. 12 pwt. and 16 gr.

9. From £59 10*s.* 7*d.* 2 far., take £37 12*s.* 9*d.* 3 far.

10. From 15 cwt. 1 gr. 18 lb. 9 oz., take 10 cwt. 21 lb. 15 oz.

11. A note was given Jan. 1, 1892, and paid Feb. 15, 1894. How long did the note run?

Ans. 2 yr. 1 mo. 14 da.

12. A note was given Oct. 21, 1890, and paid Aug. 13, 1893. How long did the note run?

13. A note, dated April 2, 1891, and paid March 1, 1892, ran how long?

14. A note was dated Dec. 21, 1892, and paid Jan. 1, 1894. Find the length of time the note ran.

15. Find the difference of time between July 4, 1776, and October 12, 1492.

16. A man was born April 2, 1853. How old was he Feb. 15, 1895?

142. Multiplication and Division.

1. Multiply £13 9*s.* 5*d.* 3 far. by 7.

PROCESS.

£	*s.*	*d.*	far.
13	9	5	3
			7
94	6	4	1

Explanation. — 7 times 3 far. are 21 far., equal to 5*d.* and 1 far.; write the 1 far. in the place of farthings, and add the 5*d.* to the product of pence; 7 times 5*d.* are 35*d.*, which, with the 5*d.* added, makes 40*d.*; 40*d.* = 3*s.* and 4*d.*; write the 4*d.* in the place of pence, and add the 3*s.* to the product of shillings; 7 times 9*s.* are 63*s.*, which, with the 3*s.* added, makes 66*s.*; 66*s.* = £3 and 6*s.*; write the 6*s.* in the place of shillings, and add £3 to the product of pounds; 7 times £13 are £91, which, with the £3 added, makes £94. Hence £13 9*s.* 5*d.* 3 far. multiplied by 7 are £94 6*s.* 4*d.* 1 far.

Rule. — *Write the multiplier under the lowest denomination of the multiplicand. Multiply as in abstract numbers; and "carry" as in the addition of compound denominate numbers.*

2. Divide £94 6*s.* 4*d.* 1 far. by 7.

PROCESS.

	£	*s.*	*d.*	far.
7	94	6	4	1
	13	9	5	3

Explanation. — $\frac{1}{7}$ of £94 is £13 with a remainder of £3 = 60*s.*; write the £13 in the place of the pounds, and add the 60*s.* to the 6*s.*; $\frac{1}{7}$ of 66*s.* is 9*s.* with a remainder of 3*s.* = 36*d.*; write the 9*s.* in the place of shillings, and add the 36*d.* to the 4*d.* = 40*d*; $\frac{1}{7}$ of 40*d.* is 5*d.* with a remainder of 5*d.*, which are equal to 20 far.; write the 5*d.* in the place of pence, and add the 20 far. to the 1 far. = 21 far.; $\frac{1}{7}$ of 21 far. is 3 far.; write the 3 far. in the place of the farthings, and the division is complete. Hence £94 6*s.* 4*d.* 1 far. divided by 7 is £13 9*s.* 5*d.* 3 far.

Rule. — *Divide the highest denomination by the divisor and reduce the remainder, if any, to the next lower denomination; add the remainder, thus reduced, to the number of this lower denomination and find the quotient of this sum. Proceed in the same way with each of the denominations to the last, and the several quotients connected together will be the entire quotient required.*

Prove as in the division of abstract numbers.

Note. — When it is desired to find how often one compound denominate number is contained in another, reduce them both to their lowest common denomination, and divide as in abstract numbers.

3. Multiply 16 bu. 3 pk. 1 qt. 1 pt. by 9.
4. Multiply 9 cwt. 15 lb. 7 oz. by 25.
5. Multiply 26 gal. 3 qt. 1 pt. 3 gi. by 12.
6. Multiply 9 mo. 25 da. 8 hr. 45 min. by 8.
7. Verify each of the foregoing.
8. Divide 28 bu. 3 pk. 7 qt. 1 pt. by 7.
9. Divide 9 cwt. 16 lb. 12 oz. by 12.
10. Divide 35 gal. 3 qt. 1 pt. 2 gi. by 9.
11. Divide 10 mo. 21 da. 15 hr. 30 min. by 6.
12. Divide £27 15*s.* 9*d.* 3 far. by 7.
13. How many bushels of wheat will 15 acres of land produce at the rate of 9 bu. 3 pk. 7 qt. 1 pt. to the acre?
14. If a field of 3 acres produce 29 bu. 3 pk. 2 qt. 1 pt. of wheat, what is the average yield per acre?
15. What will it cost to slate a roof 60 ft. long by 40 ft. wide, at $2 per 100 sq. ft.?
16. What will be the cost of 375 planks, each 15 ft. long, 18 in. wide, 2 in. thick, at $3 per 1000 board feet?

17. A farmer sold in one summer 3758 boxes of fruit averaging one quart each. How many bushels did he sell?

18. The area of a rectangular plane is 1665 sq. yd., and its length 45 yd. Find its breadth.

19. The volume of a rectangular solid is 42000 cu. yd., and the area of its base, 900 sq. yd. Find its height. What is its capacity in gallons?

20. Find the area of the base of a rectangular box whose volume is 504 cu. ft. and height 7 ft. What is its capacity in gallons?

143. Longitude and Time.

In order to reduce °, ′, ″ of longitude to hours, minutes, and seconds of time; and conversely to reduce hours, minutes, and seconds of time to degrees, minutes, and seconds of longitude, it is necessary in the first place to find the ratio between the denominations of time and the corresponding denomination of longitude.

Every point on the earth's surface, except the poles, passes through 360° in a day of 24 hours, or 15° an hour, whence,

15° of longitude = 1 hour in time.
15′ of longitude = 1 minute in time.
15″ of longitude = 1 second in time.

That is, 1 hr. 1 min. 1 sec. in time corresponds to 15° 15′ 15″ in longitude; or the longitude is to the time as 15 to 1. Hence,

To reduce Time to Longitude.

Multiply the time expressed in hours, minutes, seconds by 15.

TO REDUCE LONGITUDE TO TIME.

Divide the longitude expressed in °, ′, ″ by 15.

NOTE. — Since the sun appears to pass from east to west, when it is 12 o'clock at any given place it will be after 12 o'clock at all places east, and before 12 o'clock at all places west.

In the examples that follow, longitude is reckoned from Greenwich.

1. The city of Washington is 77° west. When it is noon at Washington, what time is it at Greenwich? When it is noon at Greenwich, what time is it at Washington?

Find the difference in time between two places when the difference in longitude is:

2. 6°, 21′.
3. 9°, 30′.
4. 8°, 45′, 12″.
5. 98°, 35′, 42″.
6. 125°, 40′, 32″.

Find the difference in longitude between two places when the difference in time is:

7. 2 hr. 30 min.
8. 5 hr. 15 min.
9. 9 hr. 45 min. 16 sec.
10. 6 hr. 15 min. 20 sec.
11. 10 hr. 37 min. 40 sec.
12. 11 hr. 11 min. 11 sec.

CHAPTER VII.

FRACTIONS.

PRIMARY METHOD.

144. A **Fraction** is one or more of the equal parts of anything.

NOTE. — This definition is merely provisional. When the fractional idea is thus concreted, each of the equal parts into which the whole is partitioned becomes a unit of a lower order; as quart, pint, and gill do in Liquid Measure. The fractional idea becomes obscure. Concreted fractions should be treated precisely as Denominate Numbers.

145. The **Unit** of the **Fraction** is the whole, which is partitioned.

146. The **Fractional Unit** is one of the equal parts into which the whole is partitioned.

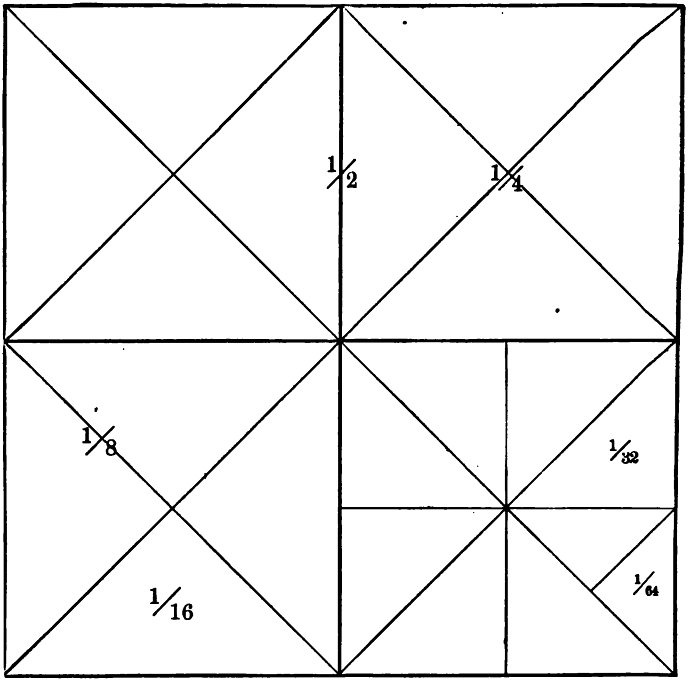
147. The **Denominator** names the fractional unit. It is strictly a *name* in the concrete method, as quarts, shillings, pounds, and should not be treated as a number.

148. The **Numerator** tells the number of fractional units taken.

149. A fraction may be written in several ways; as, five-sixths, 5–6ths, $\frac{5}{6}$.

150. The numerator and denominator taken together are called **Terms** of a fraction.

151.



NOTE.—Let the teacher be at pains to make the pupil see that twice $\frac{1}{8}$ is directly $\frac{1}{4}$ and not simply $\frac{2}{8} = \frac{1}{4}$.

152. Using the diagram D, illustrate the following:

$\frac{1}{2}$ of $1 = \frac{1}{2}$; $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$; $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$; $\frac{1}{2}$ of $\frac{1}{8} = \frac{1}{16}$; $\frac{1}{2}$ of $\frac{1}{16} = \frac{1}{32}$; $\frac{1}{2}$ of $\frac{1}{32} = \frac{1}{64}$; $2 \times \frac{1}{2} = 1$; $2 \times \frac{1}{4} = \frac{1}{2}$; $2 \times \frac{1}{8} = \frac{1}{4}$; $2 \times \frac{1}{16} = \frac{1}{8}$; $2 \times \frac{1}{32} = \frac{1}{16}$; $2 \times \frac{1}{64} = \frac{1}{32}$; $2 \times \frac{1}{128} = \frac{1}{64}$; $4 \times \frac{1}{16} = \frac{1}{4}$; $4 \times \frac{1}{32} = \frac{1}{8}$; $4 \times \frac{1}{64} = \frac{1}{16}$; $4 \times \frac{1}{128} = \frac{1}{32}$; $8 \times \frac{1}{16} = \frac{1}{2}$; $8 \times \frac{1}{64} = \frac{1}{8}$; $8 \times \frac{1}{32} = \frac{1}{4}$; $8 \times \frac{1}{128} = \frac{1}{16}$; $8 \times \frac{1}{256} = \frac{1}{32}$; etc.

Illustrate by suitable diagrams the following:

153. FRACTION TABLE.

$\frac{1}{2}\times 1=\frac{1}{2}$	$2\times\frac{1}{2}=1$	$\frac{1}{2}\times\frac{1}{7}=\frac{1}{14}$	$2\times\frac{1}{14}=\frac{1}{7}$
$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$	$2\times\frac{1}{4}=\frac{1}{2}$	$\frac{1}{2}\times\frac{1}{8}=\frac{1}{16}$	$2\times\frac{1}{16}=\frac{1}{8}$
$\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$	$2\times\frac{1}{6}=\frac{1}{3}$	$\frac{1}{2}\times\frac{1}{9}=\frac{1}{18}$	$2\times\frac{1}{18}=\frac{1}{9}$
$\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$	$2\times\frac{1}{8}=\frac{1}{4}$	$\frac{1}{2}\times\frac{1}{10}=\frac{1}{20}$	$2\times\frac{1}{20}=\frac{1}{10}$
$\frac{1}{2}\times\frac{1}{5}=\frac{1}{10}$	$2\times\frac{1}{10}=\frac{1}{5}$	$\frac{1}{2}\times\frac{1}{11}=\frac{1}{22}$	$2\times\frac{1}{22}=\frac{1}{11}$
$\frac{1}{2}\times\frac{1}{6}=\frac{1}{12}$	$2\times\frac{1}{12}=\frac{1}{6}$	$\frac{1}{2}\times\frac{1}{12}=\frac{1}{24}$	$2\times\frac{1}{24}=\frac{1}{12}$
$\frac{1}{2}\times\frac{1}{8}=\frac{1}{16}$	$2\times\frac{1}{24}=\frac{1}{12}$	$\frac{1}{2}\times\frac{1}{16}=\frac{1}{32}$	$2\times\frac{1}{36}=\frac{1}{18}$
$\frac{1}{2}\times\frac{1}{16}=\frac{1}{32}$	$2\times\frac{1}{32}=\frac{1}{16}$	$\frac{1}{2}\times\frac{1}{32}=\frac{1}{64}$	$2\times\frac{1}{48}=\frac{1}{24}$
$\frac{1}{2}\times\frac{1}{32}=\frac{1}{64}$	$2\times\frac{1}{64}=\frac{1}{32}$	$\frac{1}{2}\times\frac{1}{64}=\frac{1}{128}$	$2\times\frac{1}{96}=\frac{1}{48}$
$\frac{1}{2}\times\frac{1}{64}=\frac{1}{128}$	$2\times\frac{1}{128}=\frac{1}{64}$	$\frac{1}{2}\times\frac{1}{128}=\frac{1}{256}$	$2\times\frac{1}{192}=\frac{1}{96}$

$\frac{1}{3}\times 1=\frac{1}{3}$	$3\times\frac{1}{3}=1$	$\frac{1}{4}\times 1=\frac{1}{4}$	$4\times\frac{1}{4}=1$
$\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$	$3\times\frac{1}{6}=\frac{1}{2}$	$\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$	$4\times\frac{1}{8}=\frac{1}{2}$
$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$	$3\times\frac{1}{9}=\frac{1}{3}$	$\frac{1}{4}\times\frac{1}{3}=\frac{1}{12}$	$4\times\frac{1}{12}=\frac{1}{3}$
$\frac{1}{3}\times\frac{1}{4}=\frac{1}{12}$	$3\times\frac{1}{12}=\frac{1}{4}$	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$	$4\times\frac{1}{16}=\frac{1}{4}$
$\frac{1}{3}\times\frac{1}{5}=\frac{1}{15}$	$3\times\frac{1}{15}=\frac{1}{5}$	$\frac{1}{4}\times\frac{1}{5}=\frac{1}{20}$	$4\times\frac{1}{20}=\frac{1}{5}$
$\frac{1}{3}\times\frac{1}{6}=\frac{1}{18}$	$3\times\frac{1}{18}=\frac{1}{6}$	$\frac{1}{4}\times\frac{1}{6}=\frac{1}{24}$	$4\times\frac{1}{24}=\frac{1}{6}$
$\frac{1}{3}\times\frac{1}{7}=\frac{1}{21}$	$3\times\frac{1}{21}=\frac{1}{7}$	$\frac{1}{4}\times\frac{1}{7}=\frac{1}{28}$	$4\times\frac{1}{28}=\frac{1}{7}$
$\frac{1}{3}\times\frac{1}{8}=\frac{1}{24}$	$3\times\frac{1}{24}=\frac{1}{8}$	$\frac{1}{4}\times\frac{1}{8}=\frac{1}{32}$	$4\times\frac{1}{32}=\frac{1}{8}$
$\frac{1}{3}\times\frac{1}{9}=\frac{1}{27}$	$3\times\frac{1}{27}=\frac{1}{9}$	$\frac{1}{4}\times\frac{1}{9}=\frac{1}{36}$	$4\times\frac{1}{36}=\frac{1}{9}$
$\frac{1}{3}\times\frac{1}{10}=\frac{1}{30}$	$3\times\frac{1}{30}=\frac{1}{10}$	$\frac{1}{4}\times\frac{1}{10}=\frac{1}{40}$	$4\times\frac{1}{40}=\frac{1}{10}$
$\frac{1}{3}\times\frac{1}{11}=\frac{1}{33}$	$3\times\frac{1}{33}=\frac{1}{11}$	$\frac{1}{4}\times\frac{1}{11}=\frac{1}{44}$	$4\times\frac{1}{44}=\frac{1}{11}$
$\frac{1}{3}\times\frac{1}{12}=\frac{1}{36}$	$3\times\frac{1}{36}=\frac{1}{12}$	$\frac{1}{4}\times\frac{1}{12}=\frac{1}{48}$	$4\times\frac{1}{48}=\frac{1}{12}$
$\frac{1}{3}\times\frac{1}{16}=\frac{1}{48}$	$3\times\frac{1}{48}=\frac{1}{16}$	$\frac{1}{4}\times\frac{1}{16}=\frac{1}{64}$	$4\times\frac{1}{96}=\frac{1}{24}$
$\frac{1}{3}\times\frac{1}{24}=\frac{1}{72}$	$3\times\frac{1}{72}=\frac{1}{24}$	$\frac{1}{4}\times\frac{1}{32}=\frac{1}{128}$	$4\times\frac{1}{192}=\frac{1}{48}$

$\frac{1}{5}\times 1=\frac{1}{5}$
$\frac{1}{5}\times\frac{1}{2}=\frac{1}{10}$
$\frac{1}{5}\times\frac{1}{3}=\frac{1}{15}$
$\frac{1}{5}\times\frac{1}{4}=\frac{1}{20}$
$\frac{1}{5}\times\frac{1}{5}=\frac{1}{25}$
$\frac{1}{5}\times\frac{1}{6}=\frac{1}{30}$
$\frac{1}{5}\times\frac{1}{7}=\frac{1}{35}$
$\frac{1}{5}\times\frac{1}{8}=\frac{1}{40}$
$\frac{1}{5}\times\frac{1}{9}=\frac{1}{45}$
$\frac{1}{5}\times\frac{1}{10}=\frac{1}{50}$
$\frac{1}{5}\times\frac{1}{11}=\frac{1}{55}$
$\frac{1}{5}\times\frac{1}{12}=\frac{1}{60}$

$5\times\frac{1}{5}=1$
$5\times\frac{1}{10}=\frac{1}{2}$
$5\times\frac{1}{15}=\frac{1}{3}$
$5\times\frac{1}{20}=\frac{1}{4}$
$5\times\frac{1}{25}=\frac{1}{5}$
$5\times\frac{1}{30}=\frac{1}{6}$
$5\times\frac{1}{35}=\frac{1}{7}$
$5\times\frac{1}{40}=\frac{1}{8}$
$5\times\frac{1}{45}=\frac{1}{9}$
$5\times\frac{1}{50}=\frac{1}{10}$
$5\times\frac{1}{55}=\frac{1}{11}$
$5\times\frac{1}{60}=\frac{1}{12}$

$\frac{1}{6}\times 1=\frac{1}{6}$
$\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$
$\frac{1}{6}\times\frac{1}{3}=\frac{1}{18}$
$\frac{1}{6}\times\frac{1}{4}=\frac{1}{24}$
$\frac{1}{6}\times\frac{1}{5}=\frac{1}{30}$
$\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$
$\frac{1}{6}\times\frac{1}{7}=\frac{1}{42}$
$\frac{1}{6}\times\frac{1}{8}=\frac{1}{48}$
$\frac{1}{6}\times\frac{1}{9}=\frac{1}{54}$
$\frac{1}{6}\times\frac{1}{10}=\frac{1}{60}$
$\frac{1}{6}\times\frac{1}{11}=\frac{1}{66}$
$\frac{1}{6}\times\frac{1}{12}=\frac{1}{72}$

$6\times\frac{1}{6}=1$
$6\times\frac{1}{12}=\frac{1}{2}$
$6\times\frac{1}{18}=\frac{1}{3}$
$6\times\frac{1}{24}=\frac{1}{4}$
$6\times\frac{1}{30}=\frac{1}{5}$
$6\times\frac{1}{36}=\frac{1}{6}$
$6\times\frac{1}{42}=\frac{1}{7}$
$6\times\frac{1}{48}=\frac{1}{8}$
$6\times\frac{1}{54}=\frac{1}{9}$
$6\times\frac{1}{60}=\frac{1}{10}$
$6\times\frac{1}{66}=\frac{1}{11}$
$6\times\frac{1}{72}=\frac{1}{12}$

$\frac{1}{7}\times 1=\frac{1}{7}$
$\frac{1}{7}\times\frac{1}{2}=\frac{1}{14}$
$\frac{1}{7}\times\frac{1}{3}=\frac{1}{21}$
$\frac{1}{7}\times\frac{1}{4}=\frac{1}{28}$
$\frac{1}{7}\times\frac{1}{5}=\frac{1}{35}$
$\frac{1}{7}\times\frac{1}{6}=\frac{1}{42}$
$\frac{1}{7}\times\frac{1}{7}=\frac{1}{49}$
$\frac{1}{7}\times\frac{1}{8}=\frac{1}{56}$
$\frac{1}{7}\times\frac{1}{9}=\frac{1}{63}$
$\frac{1}{7}\times\frac{1}{10}=\frac{1}{70}$
$\frac{1}{7}\times\frac{1}{11}=\frac{1}{77}$
$=\frac{1}{84}$

$7\times\frac{1}{7}=1$
$7\times\frac{1}{14}=\frac{1}{2}$
$7\times\frac{1}{21}=\frac{1}{3}$
$7\times\frac{1}{28}=\frac{1}{4}$
$7\times\frac{1}{35}=\frac{1}{5}$
$7\times\frac{1}{42}=\frac{1}{6}$
$7\times\frac{1}{49}=\frac{1}{7}$
$7\times\frac{1}{56}=\frac{1}{8}$
$7\times\frac{1}{63}=\frac{1}{9}$
$7\times\frac{1}{70}=\frac{1}{10}$
$7\times\frac{1}{77}=\frac{1}{11}$
$7\times\frac{1}{84}=\frac{1}{12}$

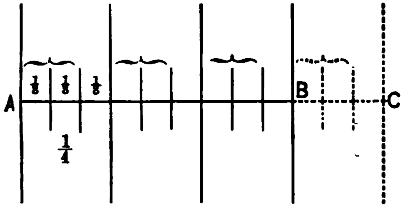
$\frac{1}{8}\times 1=\frac{1}{8}$
$\frac{1}{8}\times\frac{1}{2}=\frac{1}{16}$
$\frac{1}{8}\times\frac{1}{3}=\frac{1}{24}$
$\frac{1}{8}\times\frac{1}{4}=\frac{1}{32}$
$\frac{1}{8}\times\frac{1}{5}=\frac{1}{40}$
$\frac{1}{8}\times\frac{1}{6}=\frac{1}{48}$
$\frac{1}{8}\times\frac{1}{7}=\frac{1}{56}$
$\frac{1}{8}\times\frac{1}{8}=\frac{1}{64}$
$\frac{1}{8}\times\frac{1}{9}=\frac{1}{72}$
$\frac{1}{8}\times\frac{1}{10}=\frac{1}{80}$
$\frac{1}{8}\times\frac{1}{11}=\frac{1}{88}$
$\frac{1}{8}\times\frac{1}{12}=\frac{1}{96}$

$8\times\frac{1}{8}=1$
$8\times\frac{1}{16}=\frac{1}{2}$
$8\times\frac{1}{24}=\frac{1}{3}$
$8\times\frac{1}{32}=\frac{1}{4}$
$8\times\frac{1}{40}=\frac{1}{5}$
$8\times\frac{1}{48}=\frac{1}{6}$
$8\times\frac{1}{56}=\frac{1}{7}$
$8\times\frac{1}{64}=\frac{1}{8}$
$8\times\frac{1}{72}=\frac{1}{9}$
$8\times\frac{1}{80}=\frac{1}{10}$
$8\times\frac{1}{88}=\frac{1}{11}$
$8\times\frac{1}{96}=\frac{1}{12}$

$\frac{1}{9} \times 1 = \frac{1}{9}$	$9 \times \frac{1}{9} = 1$	$\frac{1}{10} \times 1 = \frac{1}{10}$	$10 \times \frac{1}{10} = 1$
$\frac{1}{9} \times \frac{1}{2} = \frac{1}{18}$	$9 \times \frac{1}{18} = \frac{1}{2}$	$\frac{1}{10} \times \frac{1}{2} = \frac{1}{20}$	$10 \times \frac{1}{20} = \frac{1}{2}$
$\frac{1}{9} \times \frac{1}{3} = \frac{1}{27}$	$9 \times \frac{1}{27} = \frac{1}{3}$	$\frac{1}{10} \times \frac{1}{3} = \frac{1}{30}$	$10 \times \frac{1}{30} = \frac{1}{3}$
$\frac{1}{9} \times \frac{1}{4} = \frac{1}{36}$	$9 \times \frac{1}{36} = \frac{1}{4}$	$\frac{1}{10} \times \frac{1}{4} = \frac{1}{40}$	$10 \times \frac{1}{40} = \frac{1}{4}$
$\frac{1}{9} \times \frac{1}{5} = \frac{1}{45}$	$9 \times \frac{1}{45} = \frac{1}{5}$	$\frac{1}{10} \times \frac{1}{5} = \frac{1}{50}$	$10 \times \frac{1}{50} = \frac{1}{5}$
$\frac{1}{9} \times \frac{1}{6} = \frac{1}{54}$	$9 \times \frac{1}{54} = \frac{1}{6}$	$\frac{1}{10} \times \frac{1}{6} = \frac{1}{60}$	$10 \times \frac{1}{60} = \frac{1}{6}$
$\frac{1}{9} \times \frac{1}{7} = \frac{1}{63}$	$9 \times \frac{1}{63} = \frac{1}{7}$	$\frac{1}{10} \times \frac{1}{7} = \frac{1}{70}$	$10 \times \frac{1}{70} = \frac{1}{7}$
$\frac{1}{9} \times \frac{1}{8} = \frac{1}{72}$	$9 \times \frac{1}{72} = \frac{1}{8}$	$\frac{1}{10} \times \frac{1}{8} = \frac{1}{80}$	$10 \times \frac{1}{80} = \frac{1}{8}$
$\frac{1}{9} \times \frac{1}{9} = \frac{1}{81}$	$9 \times \frac{1}{81} = \frac{1}{9}$	$\frac{1}{10} \times \frac{1}{9} = \frac{1}{90}$	$10 \times \frac{1}{90} = \frac{1}{9}$
$\frac{1}{9} \times \frac{1}{10} = \frac{1}{90}$	$9 \times \frac{1}{90} = \frac{1}{10}$	$\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$	$10 \times \frac{1}{100} = \frac{1}{10}$
$\frac{1}{9} \times \frac{1}{11} = \frac{1}{99}$	$9 \times \frac{1}{99} = \frac{1}{11}$	$\frac{1}{10} \times \frac{1}{11} = \frac{1}{110}$	$10 \times \frac{1}{110} = \frac{1}{11}$
$\frac{1}{9} \times \frac{1}{12} = \frac{1}{108}$	$9 \times \frac{1}{108} = \frac{1}{12}$	$\frac{1}{10} \times \frac{1}{12} = \frac{1}{120}$	$10 \times \frac{1}{120} = \frac{1}{12}$

$\frac{1}{11} \times 1 = \frac{1}{11}$	$11 \times \frac{1}{11} = 1$	$\frac{1}{12} \times 1 = \frac{1}{12}$	$12 \times \frac{1}{12} = 1$
$\frac{1}{11} \times \frac{1}{2} = \frac{1}{22}$	$11 \times \frac{1}{22} = \frac{1}{2}$	$\frac{1}{12} \times \frac{1}{2} = \frac{1}{24}$	$12 \times \frac{1}{24} = \frac{1}{2}$
$\frac{1}{11} \times \frac{1}{3} = \frac{1}{33}$	$11 \times \frac{1}{33} = \frac{1}{3}$	$\frac{1}{12} \times \frac{1}{3} = \frac{1}{36}$	$12 \times \frac{1}{36} = \frac{1}{3}$
$\frac{1}{11} \times \frac{1}{4} = \frac{1}{44}$	$11 \times \frac{1}{44} = \frac{1}{4}$	$\frac{1}{12} \times \frac{1}{4} = \frac{1}{48}$	$12 \times \frac{1}{48} = \frac{1}{4}$
$\frac{1}{11} \times \frac{1}{5} = \frac{1}{55}$	$11 \times \frac{1}{55} = \frac{1}{5}$	$\frac{1}{12} \times \frac{1}{5} = \frac{1}{60}$	$12 \times \frac{1}{60} = \frac{1}{5}$
$\frac{1}{11} \times \frac{1}{6} = \frac{1}{66}$	$11 \times \frac{1}{66} = \frac{1}{6}$	$\frac{1}{12} \times \frac{1}{6} = \frac{1}{72}$	$12 \times \frac{1}{72} = \frac{1}{6}$
$\frac{1}{11} \times \frac{1}{7} = \frac{1}{77}$	$11 \times \frac{1}{77} = \frac{1}{7}$	$\frac{1}{12} \times \frac{1}{7} = \frac{1}{84}$	$12 \times \frac{1}{84} = \frac{1}{7}$
$\frac{1}{11} \times \frac{1}{8} = \frac{1}{88}$	$11 \times \frac{1}{88} = \frac{1}{8}$	$\frac{1}{12} \times \frac{1}{8} = \frac{1}{96}$	$12 \times \frac{1}{96} = \frac{1}{8}$
$\frac{1}{11} \times \frac{1}{9} = \frac{1}{99}$	$11 \times \frac{1}{99} = \frac{1}{9}$	$\frac{1}{12} \times \frac{1}{9} = \frac{1}{108}$	$12 \times \frac{1}{108} = \frac{1}{9}$
$\frac{1}{11} \times \frac{1}{10} = \frac{1}{110}$	$11 \times \frac{1}{110} = \frac{1}{10}$	$\frac{1}{12} \times \frac{1}{10} = \frac{1}{120}$	$12 \times \frac{1}{120} = \frac{1}{10}$
$\frac{1}{11} \times \frac{1}{11} = \frac{1}{121}$	$11 \times \frac{1}{121} = \frac{1}{11}$	$\frac{1}{12} \times \frac{1}{11} = \frac{1}{132}$	$12 \times \frac{1}{132} = \frac{1}{11}$
$\frac{1}{11} \times \frac{1}{12} = \frac{1}{132}$	$11 \times \frac{1}{132} = \frac{1}{12}$	$\frac{1}{12} \times \frac{1}{12} = \frac{1}{144}$	$12 \times \frac{1}{144} = \frac{1}{12}$

154. Take $\frac{2}{3}$ times $\frac{3}{4}$.



OPERATION (*a*).

$\frac{1}{3}$ of 3–4ths = 1–4th.
$\frac{2}{3}$ of 3–4ths = 2–4ths.

OPERATION (*b*).

$\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$
$\frac{2}{3}$ of $\frac{1}{4} = \frac{2}{12}$
$\frac{2}{3}$ of $\frac{3}{4} = \frac{6}{12}$

OPERATION (*b*). *See Diagram.*

In the above diagram the line *AC* is divided into 4 equal parts, of which the dotted line *BC* represents the missing 4th.

Now divide each of the 3 fourths into 3 equal parts, and take 2 from each of them as indicated by the braces.

We shall then have 6 parts, and the parts will be 12ths; for, if each of the 4 fourths be divided into equal parts, the whole (or *AC*) will be divided into 12 equal parts. Hence $\frac{2}{3}$ of $\frac{3}{4} = \frac{6}{12}$.

Illustrate the following:

1. $\frac{2}{3}$ of $\frac{1}{2}$ is what part of 1?
2. $\frac{3}{4}$ times $\frac{1}{2}$ is what part of 1?
3. $\frac{2}{3}$ of $\frac{5}{6}$ is what part of 1?
4. $\frac{3}{4}$ of $\frac{5}{6}$ is what part of 1?
5. $\frac{1}{2}$ of $\frac{2}{3}$ is what part of 1?
6. $\frac{1}{2}$ of $\frac{3}{4}$ is what part of 1?
7. $\frac{1}{3}$ of $\frac{3}{4}$ is what part of 1?
8. $\frac{1}{5}$ of $\frac{5}{6}$ is what part of 1?
9. $\frac{2}{3}$ of $\frac{1}{5}$ is what part of 1?
10. $\frac{3}{4}$ of $\frac{1}{3}$ is what part of 1?
11. $\frac{5}{6}$ of $\frac{1}{4}$ is what part of 1?
12. $\frac{2}{3}$ of $\frac{1}{8}$ is what part of 1?
13. $\frac{3}{4}$ of $\frac{5}{8}$ is what part of 1?
14. 1 pt. is what part of a gal.?

155. A **Proper Fraction** is one whose value is less than the unit of the fraction; consequently its numerator is less than its denominator; as, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.

156. An **Improper Fraction** is one whose value is equal to or greater than the unit of the fraction; consequently its numerator is equal to or greater than its denominator; as, $\frac{4}{4}$, $\frac{5}{4}$, $\frac{7}{5}$.

157. A **Mixed Number** is an integer and a fraction; as $2\frac{1}{2}$, $5\frac{3}{4}$.

158. A **Simple Fraction** is a single fraction whose numerator and denominator are both integers; as $\frac{1}{2}$, $\frac{7}{8}$.

159. A **Compound Fraction** is a fraction of a fraction; as $\frac{2}{3}$ of $\frac{3}{4}$; $\frac{5}{6} \times \frac{8}{9}$.

160. A **Complex Fraction** is one whose numerator or denominator or both, are fractions; as, $\frac{2}{\frac{3}{4}}$, $\frac{\frac{2}{3}}{4}$, $\frac{\frac{2}{3}}{\frac{3}{4}}$.

161. The **Denomination** of a fraction is the *order* of its fractional unit.

162. **Reduction** of a fraction is the process of changing its denomination without altering its value.

163. **Reduction Ascending** is the process of changing a fraction to a higher denomination.

164. **Reduction Descending** is the process of changing a fraction to a lower denomination.

REDUCTION.

Case I.

165. To reduce fractions to lower denominations (higher terms).

1. Reduce $\frac{5}{6}$ to 18ths.

OPERATION.

$1 = \frac{18}{18}$.

$\frac{1}{6} = \frac{3}{18}$; therefore

$\frac{5}{6} = 5$ times $\frac{3}{18} = \frac{15}{18}$.

Or,

$\frac{5}{6} = 3$ times $\frac{5}{18} = \frac{15}{18}$.

EXPLANATION. — 1-6th = 3-18ths. 5-6ths = 5 times 3-18ths = 15-18ths: or, since the ratio of 6ths to 18ths is 3, there are 3 times as many 18ths as 6ths in any number; therefore 5-6ths is equal to 3 times 5-18ths, which is 15-18ths.

Reduce the following:

2. $\frac{3}{4}$ to 12ths.
3. $\frac{5}{6}$ to 18ths.
4. $\frac{5}{6}$ to 24ths.
5. $\frac{5}{6}$ to 30ths.
6. $\frac{2}{3}$ to 9ths.
7. $\frac{2}{3}$ to 15ths.
8. $\frac{2}{3}$ to 21sts.
9. $\frac{1}{2}$ to 40ths.
10. $\frac{1}{2}$ to 20ths.
11. $\frac{1}{2}$ to 80ths.
12. $\frac{7}{8}$ to 64ths.
13. $\frac{7}{8}$ to 32ds.
14. $\frac{7}{8}$ to 16ths.
15. $\frac{5}{7}$ to 14ths.
16. $\frac{5}{7}$ to 42ds.
17. $\frac{5}{7}$ to 84ths.
18. $\frac{3}{7}$ to 21sts.
19. $\frac{5}{9}$ to 18ths.
20. $\frac{5}{9}$ to 36ths.
21. $\frac{5}{9}$ to 72ds.
22. $\frac{3}{10}$ to 30ths.
23. $\frac{3}{10}$ to 60ths.
24. $\frac{3}{5}$ to 25ths.
25. $\frac{7}{9}$ to 63ds.
26. $\frac{5}{8}$ to 48ths.
27. $\frac{5}{12}$ to 108ths.
28. $\frac{5}{12}$ to 96ths.
29. $\frac{7}{11}$ to 22ds.
30. $\frac{7}{11}$ to 44ths.
31. $\frac{17}{35}$ to 70ths.
32. $\frac{13}{32}$ to 64ths.
33. $\frac{11}{24}$ to 32ds.
34. $\frac{11}{13}$ to 39ths.
35. $\frac{15}{16}$ to 64ths.
36. $\frac{14}{15}$ to 60ths.
37. $\frac{13}{17}$ to 34ths.
38. $\frac{11}{18}$ to 36ths.
39. $\frac{20}{21}$ to 84ths.
40. $\frac{17}{33}$ to 99ths.
41. $\frac{19}{27}$ to 81sts.

166. To reduce fractions to higher denominations (lower terms).

Case II.

1. Reduce $\frac{12}{18}$ to its highest denomination (lowest terms).

OPERATION.

6–18ths = 1–3d;
$12 \div 6 = 2$;
therefore $\frac{12}{18} = \frac{2}{3}$.
Or,
$12 \div 3 = 4$;
therefore $\frac{12}{18} = \frac{4}{6}$.
$4 \div 2 = 2$;
therefore $\frac{4}{6} = \frac{2}{3}$.

EXPLANATION. — Since 2–18ths make 1–9th, 3–18ths make 1–6th, 6–18ths make 1–3d, and 12 is a multiple of 2, of 3, and of 6; 12–18ths is reducible to 9ths, 6ths, and 3ds. But 3ds is the highest denomination. 6–18ths make a 3d: therefore 12–18ths make as many 3ds as 6 is contained times in 12, which is 2. Whence $\frac{12}{18} = \frac{2}{3}$; or 12–18ths may be reduced to 6ths and the 6ths to 3ds.

Reduce the following to their highest denominations, and verify by reduction back to lower denominations by preceding case:

2. $\frac{9}{12}$.
3. $\frac{8}{12}$.
4. $\frac{6}{12}$.
5. $\frac{10}{12}$.
6. $\frac{12}{12}$.
7. $\frac{24}{12}$.
8. $\frac{36}{12}$.
9. $\frac{48}{12}$.
10. $\frac{60}{12}$.
11. $\frac{6}{8}$.
12. $\frac{8}{10}$.
13. $\frac{5}{15}$.
14. $\frac{10}{15}$.
15. $\frac{15}{18}$.
16. $\frac{12}{16}$.
17. $\frac{15}{20}$.
18. $\frac{21}{18}$.
19. $\frac{24}{20}$.
20. $\frac{36}{48}$.
21. $\frac{72}{108}$.
22. $\frac{30}{45}$.
23. $\frac{25}{75}$.
24. $\frac{19}{38}$.
25. $\frac{35}{215}$.
26. $\frac{27}{81}$.
27. $\frac{48}{64}$.
28. $\frac{70}{80}$.
29. $\frac{75}{100}$.
30. $\frac{20}{100}$.
31. $\frac{40}{100}$.
32. $\frac{50}{100}$.
33. $\frac{60}{100}$.
34. $\frac{45}{100}$.
35. $\frac{84}{96}$.
36. $\frac{33}{88}$.
37. $\frac{110}{121}$.
38. $\frac{132}{144}$.
39. $\frac{45}{50}$.
40. $\frac{35}{55}$.
41. $\frac{49}{56}$.
42. $\frac{54}{63}$.
43. $\frac{72}{81}$.
44. $\frac{90}{108}$.
45. $\frac{56}{64}$.
46. $\frac{48}{56}$.
47. $\frac{40}{48}$.
48. $\frac{30}{54}$.
49. $\frac{21}{63}$.
50. $\frac{24}{51}$.

Note.—The two cases above, the one in Reduction Descending and the other in Reduction Ascending, embrace all possible reductions of fractions. Let the teacher see to it that the pupil properly class the special cases that follow under one or the other of these heads.

167. To reduce integers and mixed numbers to improper fractions.

1. Reduce $15\frac{3}{4}$ to an improper fraction.

OPERATION.

$1 = \frac{4}{4}$;
$15 = 15 \times \frac{4}{4} = \frac{60}{4}$;
$\frac{60}{4} + \frac{3}{4} = \frac{63}{4}$.

EXPLANATION.—It is required to express the integral part of the mixed number in terms of the fractional unit, and to combine it with the fractional part in one expression. The first step involves reduction descending and falls under Case I; in the second step the two expressions are merely combined in *one*. Since in one whole there are $\frac{4}{4}$, in 15 there are 15 times $\frac{4}{4}$, which is $\frac{60}{4}$; $\frac{60}{4}$ and $\frac{3}{4} = \frac{63}{4}$. Whence $15\frac{3}{4} = \frac{63}{4}$.

Reduce the following to improper fractions:

2. $3\frac{1}{3}$.
3. $5\frac{1}{2}$.
4. $6\frac{1}{4}$.
5. $8\frac{3}{5}$.
6. $9\frac{2}{3}$.
7. $10\frac{3}{4}$.
8. $25\frac{5}{6}$.
9. $36\frac{6}{7}$.
10. $47\frac{5}{9}$.
11. $42\frac{8}{9}$.
12. $59\frac{7}{8}$.
13. $67\frac{9}{10}$.
14. $84\frac{3}{8}$.
15. $72\frac{5}{12}$.
16. $91\frac{4}{5}$.
17. $99\frac{7}{11}$.

18. $125\frac{3}{8}$.
19. $175\frac{3}{25}$.
20. $93\frac{10}{11}$.
21. $97\frac{5}{18}$.
22. $136\frac{17}{19}$.
23. $145\frac{20}{21}$.
24. $376\frac{19}{26}$.
25. $487\frac{12}{25}$.
26. $589\frac{13}{45}$.
27. $695\frac{37}{43}$.
28. $701\frac{23}{37}$.
29. $864\frac{71}{99}$.
30. $729\frac{15}{16}$.
31. $339\frac{13}{33}$.
32. $558\frac{11}{18}$.
33. $776\frac{15}{16}$.
34. $999\frac{19}{25}$.
35. $683\frac{37}{45}$.
36. $258\frac{13}{56}$.
37. $273\frac{37}{100}$.
38. $489\frac{49}{100}$.
39. $965\frac{59}{66}$.
40. $736\frac{25}{31}$.
41. $224\frac{37}{45}$.
42. $497\frac{31}{49}$.
43. $735\frac{53}{63}$.
44. $765\frac{21}{45}$.
45. $873\frac{15}{18}$.

168. To reduce improper fractions to integral or mixed numbers.

1. Reduce $\frac{63}{4}$ to a mixed number.

OPERATION.

$1 = \frac{4}{4}$;

$63 \div 4 = 15\frac{3}{4}$.

EXPLANATION. — It is required to reduce $\frac{63}{4}$ to 1's and a fraction. This involves reduction ascending and falls under Case II. Since $\frac{4}{4}$ makes a whole, there are as many wholes in $\frac{63}{4}$ as 4 is contained times in 63, which is $15\frac{3}{4}$. Whence $\frac{63}{4} = 15\frac{3}{4}$. Or, since there are in any expression 1–4th as many 1's as there are 4ths, in 63–4ths there are $\frac{1}{4}$ of 63 1's, which is $15\frac{3}{4}$.

Reduce the following to integral or mixed numbers, and verify by reduction back to improper fractions by the preceding case:

2. $\frac{8}{2}$.
3. $\frac{12}{4}$.
4. $\frac{12}{3}$.
5. $\frac{12}{6}$.
6. $\frac{12}{2}$.
7. $\frac{15}{3}$.
8. $\frac{18}{3}$.
$\frac{15}{5}$.
10. $\frac{18}{6}$.
11. $\frac{18}{9}$.
12. $\frac{20}{4}$.
13. $\frac{20}{10}$.
14. $\frac{20}{20}$.
15. $\frac{20}{2}$.
16. $\frac{20}{5}$.
17. $\frac{25}{4}$.
18. $\frac{9}{2}$.
19. $\frac{6}{4}$.
20. $\frac{9}{5}$.
21. $\frac{27}{5}$.
22. $\frac{29}{7}$.
23. $\frac{31}{8}$.
24. $\frac{36}{5}$.
25. $\frac{18}{4}$.
26. $\frac{59}{17}$.
27. $\frac{62}{28}$.
28. $\frac{69}{32}$.
29. $\frac{96}{25}$.
30. $\frac{89}{42}$.
31. $\frac{72}{48}$.
32. $\frac{85}{89}$.
33. $\frac{3290}{120}$.
34. $\frac{4273}{133}$.
35. $\frac{6425}{195}$.
36. $\frac{7964}{76}$.
37. $\frac{8472}{825}$.
38. $\frac{9604}{136}$.
39. $\frac{2568}{228}$.
40. $\frac{3659}{872}$.
41. $\frac{9264}{575}$.

169. To reduce dissimilar fractions to similar fractions.

Fractions that have the same unit of the fraction and the same fractional unit, are said to be **Similar**; otherwise, they are **Dissimilar.**

Similar fractions have, of course, a Common Denominator; and when similar fractions are expressed in terms of their highest common fractional unit, they are said to have their Lowest, or Least Common Denominator.

1. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ to similar fractions.

OPERATION.

$1 = \frac{12}{12}$,
$\frac{1}{2} = \frac{1}{2}$ of $\frac{12}{12} = \frac{6}{12}$.
$1 = \frac{12}{12}$,
$\frac{1}{3} = \frac{1}{3}$ of $\frac{12}{12} = \frac{4}{12}$,
$\frac{2}{3} = 2 \times \frac{4}{12} = \frac{8}{12}$.
$1 = \frac{12}{12}$,
$\frac{1}{4} = \frac{1}{4}$ of $\frac{12}{12} = \frac{3}{12}$,
$\frac{3}{4} = 3 \times \frac{3}{12} = \frac{9}{12}$.
$1 = \frac{12}{12}$,
$\frac{1}{6} = \frac{1}{6}$ of $\frac{12}{12} = \frac{2}{12}$,
$\frac{5}{6} = 5 \times \frac{2}{12} = \frac{10}{12}$.

EXPLANATION. — These fractions are severally reducible: $\frac{1}{2}$ to 4ths, 6ths, 8ths, 10ths, 12ths, 24ths, 48ths, etc.; $\frac{2}{3}$ to 6ths, 9ths, 12ths, 24ths, 48ths, etc.; $\frac{3}{4}$ to 8ths, 12ths, 24ths, 48ths, etc.; $\frac{5}{6}$ to 12ths, 18ths, 24ths, 48ths, etc.

From which it will be seen that these fractions may all be reduced to 12ths, 24ths, 48ths. Selecting 12ths as the highest common fractional unit, or the least common denominator, we reduce as in Case I., and get $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$, and $\frac{10}{12}$ as the required fractions.

Reduce the following to similar fractions, and verify each reduction by its converse process.

NOTE. — Mixed numbers must first be reduced to improper fractions.

2. $\frac{1}{2}$, $\frac{1}{6}$.
3. $\frac{1}{2}$, $\frac{1}{4}$.
4. $\frac{1}{2}$, $\frac{1}{8}$.
5. $\frac{1}{2}$, $\frac{1}{10}$.
6. $\frac{1}{3}$, $\frac{1}{9}$.
7. $\frac{1}{2}$, $\frac{3}{8}$.
8. $\frac{2}{3}$, $\frac{3}{4}$.
9. $\frac{3}{4}$, $\frac{5}{8}$.
10. $\frac{3}{4}$, $\frac{5}{6}$.
11. $\frac{5}{6}$, $\frac{2}{3}$.
12. $\frac{3}{8}$, $\frac{5}{16}$.
13. $\frac{3}{5}$, $\frac{7}{10}$.
14. $\frac{5}{7}$, $\frac{20}{21}$.
15. $\frac{8}{9}$, $\frac{17}{18}$.
16. $\frac{9}{10}$, $\frac{21}{40}$.
17. $\frac{15}{16}$, $\frac{37}{48}$.
18. $\frac{19}{32}$, $\frac{23}{24}$.
19. $\frac{29}{30}$, $\frac{59}{60}$.

20. [illegible]
21. [illegible]
22. [illegible]
23. [illegible]
24. [illegible]
25. [illegible]

26. [illegible]
27. [illegible]
28. [illegible]
29. [illegible]
30. [illegible]
31. [illegible]

32. [illegible], $\frac{11}{12}$.
33. [illegible]
34. 2[illegible], 3[illegible].
35. 5[illegible], 6[illegible].
36. 8[illegible], 19[illegible].
37. 12[illegible], 8[illegible].

170. ADDITION AND SUBTRACTION.

Similar fractions only can be added or subtracted.

Find the sum of [illegible].

SOLUTION.

[illegible]

EXPLANATION.—Since only fractions having a common denominator can be added or subtracted, we reduce these fractions to similar fractions [illegible]

Whence the sum is [illegible].

Perform the indicated operations upon the following numbers and verify.

NOTE.—Reduce the mixed numbers to improper fractions.

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]
5. [illegible]
6. [illegible]
7. [illegible]

8. [illegible]
9. [illegible]
10. [illegible]
11. [illegible]
12. [illegible]
13. [illegible]
14. [illegible]

15. [illegible]
16. [illegible]
17. [illegible]
18. [illegible]
19. [illegible]
20. [illegible]
21. [illegible]

22. [illegible]
23. [illegible]
24. [illegible]
25. [illegible]

26. [illegible]
27. [illegible]
28. [illegible]
29. [illegible]

171. Where mixed numbers are large, addition or subtraction of them is more readily performed without reducing them to improper fractions.

Find the sum of $245\frac{14}{15}$ and $176\frac{5}{9}$.

OPERATION.

$$\frac{14}{15}+\frac{5}{9}=\frac{42}{45}+\frac{25}{45}=\frac{67}{45}=1\frac{22}{45}.$$

$$\begin{array}{r} 245 \\ 176 \\ 1\frac{22}{45} \\ \hline 422\frac{22}{45} \end{array}$$

Explanation. — Add the fractions separately; and if the result be an improper fraction, reduce it to a whole or mixed number. Set down the fractional part to the right of the units' place, and carry the integral part to the units' column, and proceed as in the addition of whole numbers.

Find the difference between $45\frac{2}{3}$ and $12\frac{3}{4}$.

OPERATION.

$$\begin{array}{ccccc} 1. & & 2. & & 3. \\ 45\frac{2}{3} & = & 45\frac{8}{12} & = & 44\frac{20}{12} \\ \underline{12\frac{3}{4}} & = & \underline{12\frac{9}{12}} & = & \underline{12\frac{9}{12}} \\ & & 32\frac{11}{12} & & 32\frac{11}{12} \end{array}$$

Explanation. — The principle is sufficiently well indicated in the 3d process. Since $\frac{9}{12}$ cannot be taken from $\frac{8}{12}$, "borrow" 1 unit from the 5 units of the minuend, reduce it to 12ths, add to 8-12ths, which give $\frac{20}{12}$; $\frac{20}{12}-\frac{9}{12}=\frac{11}{12}$; 2 from 4 leaves 2: set down the 2 under the units' figure, and proceed as in subtraction of integers.

Perform the indicated operations upon the following numbers and verify:

1. $675\frac{5}{9}+363\frac{4}{5}$.
2. $937\frac{15}{16}+789\frac{17}{24}$.
3. $845\frac{15}{21}-692\frac{6}{7}$.
4. $535-348\frac{13}{17}$.
5. $983-854\frac{11}{28}$.
6. $734\frac{5}{12}+725\frac{13}{24}$.
7. $357\frac{8}{9}+631\frac{7}{18}$.
8. $654\frac{9}{10}+359\frac{3}{5}$.
9. $793\frac{13}{27}+456\frac{5}{9}$.
10. $912\frac{3}{8}+263\frac{1}{5}$.
11. $530\frac{7}{11}-250\frac{17}{22}$.
12. $591\frac{5}{9}+965\frac{3}{8}$.
13. $832\frac{5}{16}+434\frac{3}{8}$.
14. $947\frac{17}{32}+300\frac{29}{72}$.

$\frac{1}{5}\times 1=\frac{1}{5}$
$\frac{1}{5}\times\frac{1}{2}=\frac{1}{10}$
$\frac{1}{5}\times\frac{1}{3}=\frac{1}{15}$
$\frac{1}{5}\times\frac{1}{4}=\frac{1}{20}$
$\frac{1}{5}\times\frac{1}{5}=\frac{1}{25}$
$\frac{1}{5}\times\frac{1}{6}=\frac{1}{30}$
$\frac{1}{5}\times\frac{1}{7}=\frac{1}{35}$
$\frac{1}{5}\times\frac{1}{8}=\frac{1}{40}$
$\frac{1}{5}\times\frac{1}{9}=\frac{1}{45}$
$\frac{1}{5}\times\frac{1}{10}=\frac{1}{50}$
$\frac{1}{5}\times\frac{1}{11}=\frac{1}{55}$
$\frac{1}{5}\times\frac{1}{12}=\frac{1}{60}$

$5\times\frac{1}{5}=1$
$5\times\frac{1}{10}=\frac{1}{2}$
$5\times\frac{1}{15}=\frac{1}{3}$
$5\times\frac{1}{20}=\frac{1}{4}$
$5\times\frac{1}{25}=\frac{1}{5}$
$5\times\frac{1}{30}=\frac{1}{6}$
$5\times\frac{1}{35}=\frac{1}{7}$
$5\times\frac{1}{40}=\frac{1}{8}$
$5\times\frac{1}{45}=\frac{1}{9}$
$5\times\frac{1}{50}=\frac{1}{10}$
$5\times\frac{1}{55}=\frac{1}{11}$
$5\times\frac{1}{60}=\frac{1}{12}$

$\frac{1}{6}\times 1=\frac{1}{6}$
$\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$
$\frac{1}{6}\times\frac{1}{3}=\frac{1}{18}$
$\frac{1}{6}\times\frac{1}{4}=\frac{1}{24}$
$\frac{1}{6}\times\frac{1}{5}=\frac{1}{30}$
$\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$
$\frac{1}{6}\times\frac{1}{7}=\frac{1}{42}$
$\frac{1}{6}\times\frac{1}{8}=\frac{1}{48}$
$\frac{1}{6}\times\frac{1}{9}=\frac{1}{54}$
$\frac{1}{6}\times\frac{1}{10}=\frac{1}{60}$
$\frac{1}{6}\times\frac{1}{11}=\frac{1}{66}$
$\frac{1}{6}\times\frac{1}{12}=\frac{1}{72}$

$6\times\frac{1}{6}=1$
$6\times\frac{1}{12}=\frac{1}{2}$
$6\times\frac{1}{18}=\frac{1}{3}$
$6\times\frac{1}{24}=\frac{1}{4}$
$6\times\frac{1}{30}=\frac{1}{5}$
$6\times\frac{1}{36}=\frac{1}{6}$
$6\times\frac{1}{42}=\frac{1}{7}$
$6\times\frac{1}{48}=\frac{1}{8}$
$6\times\frac{1}{54}=\frac{1}{9}$
$6\times\frac{1}{60}=\frac{1}{10}$
$6\times\frac{1}{66}=\frac{1}{11}$
$6\times\frac{1}{72}=\frac{1}{12}$

$\frac{1}{7}\times 1=\frac{1}{7}$
$\frac{1}{7}\times\frac{1}{2}=\frac{1}{14}$
$\frac{1}{7}\times\frac{1}{3}=\frac{1}{21}$
$\frac{1}{7}\times\frac{1}{4}=\frac{1}{28}$
$\frac{1}{7}\times\frac{1}{5}=\frac{1}{35}$
$\frac{1}{7}\times\frac{1}{6}=\frac{1}{42}$
$\frac{1}{7}\times\frac{1}{7}=\frac{1}{49}$
$\frac{1}{7}\times\frac{1}{8}=\frac{1}{56}$
$\frac{1}{7}\times\frac{1}{9}=\frac{1}{63}$
$\frac{1}{7}\times\frac{1}{10}=\frac{1}{70}$
$\frac{1}{7}\times\frac{1}{11}=\frac{1}{77}$
$\frac{1}{7}\times\frac{1}{12}=\frac{1}{84}$

$7\times\frac{1}{7}=1$
$7\times\frac{1}{14}=\frac{1}{2}$
$7\times\frac{1}{21}=\frac{1}{3}$
$7\times\frac{1}{28}=\frac{1}{4}$
$7\times\frac{1}{35}=\frac{1}{5}$
$7\times\frac{1}{42}=\frac{1}{6}$
$7\times\frac{1}{49}=\frac{1}{7}$
$7\times\frac{1}{56}=\frac{1}{8}$
$7\times\frac{1}{63}=\frac{1}{9}$
$7\times\frac{1}{70}=\frac{1}{10}$
$7\times\frac{1}{77}=\frac{1}{11}$
$7\times\frac{1}{84}=\frac{1}{12}$

$\frac{1}{8}\times 1=\frac{1}{8}$
$\frac{1}{8}\times\frac{1}{2}=\frac{1}{16}$
$\frac{1}{8}\times\frac{1}{3}=\frac{1}{24}$
$\frac{1}{8}\times\frac{1}{4}=\frac{1}{32}$
$\frac{1}{8}\times\frac{1}{5}=\frac{1}{40}$
$\frac{1}{8}\times\frac{1}{6}=\frac{1}{48}$
$\frac{1}{8}\times\frac{1}{7}=\frac{1}{56}$
$\frac{1}{8}\times\frac{1}{8}=\frac{1}{64}$
$\frac{1}{8}\times\frac{1}{9}=\frac{1}{72}$
$\frac{1}{8}\times\frac{1}{10}=\frac{1}{80}$
$\frac{1}{8}\times\frac{1}{11}=\frac{1}{88}$
$\frac{1}{8}\times\frac{1}{12}=\frac{1}{96}$

$8\times\frac{1}{8}=1$
$8\times\frac{1}{16}=\frac{1}{2}$
$8\times\frac{1}{24}=\frac{1}{3}$
$8\times\frac{1}{32}=\frac{1}{4}$
$8\times\frac{1}{40}=\frac{1}{5}$
$8\times\frac{1}{48}=\frac{1}{6}$
$8\times\frac{1}{56}=\frac{1}{7}$
$8\times\frac{1}{64}=\frac{1}{8}$
$8\times\frac{1}{72}=\frac{1}{9}$
$8\times\frac{1}{80}=\frac{1}{10}$
$8\times\frac{1}{88}=\frac{1}{11}$
$8\times\frac{1}{96}=\frac{1}{12}$

$\frac{1}{9}\times 1=\frac{1}{9}$	$9\times\frac{1}{9}=1$	$\frac{1}{10}\times 1=\frac{1}{10}$	$10\times\frac{1}{10}=1$
$\frac{1}{9}\times\frac{1}{2}=\frac{1}{18}$	$9\times\frac{1}{18}=\frac{1}{2}$	$\frac{1}{10}\times\frac{1}{2}=\frac{1}{20}$	$10\times\frac{1}{20}=\frac{1}{2}$
$\frac{1}{9}\times\frac{1}{3}=\frac{1}{27}$	$9\times\frac{1}{27}=\frac{1}{3}$	$\frac{1}{10}\times\frac{1}{3}=\frac{1}{30}$	$10\times\frac{1}{30}=\frac{1}{3}$
$\frac{1}{9}\times\frac{1}{4}=\frac{1}{36}$	$9\times\frac{1}{36}=\frac{1}{4}$	$\frac{1}{10}\times\frac{1}{4}=\frac{1}{40}$	$10\times\frac{1}{40}=\frac{1}{4}$
$\frac{1}{9}\times\frac{1}{5}=\frac{1}{45}$	$9\times\frac{1}{45}=\frac{1}{5}$	$\frac{1}{10}\times\frac{1}{5}=\frac{1}{50}$	$10\times\frac{1}{50}=\frac{1}{5}$
$\frac{1}{9}\times\frac{1}{6}=\frac{1}{54}$	$9\times\frac{1}{54}=\frac{1}{6}$	$\frac{1}{10}\times\frac{1}{6}=\frac{1}{60}$	$10\times\frac{1}{60}=\frac{1}{6}$
$\frac{1}{9}\times\frac{1}{7}=\frac{1}{63}$	$9\times\frac{1}{63}=\frac{1}{7}$	$\frac{1}{10}\times\frac{1}{7}=\frac{1}{70}$	$10\times\frac{1}{70}=\frac{1}{7}$
$\frac{1}{9}\times\frac{1}{8}=\frac{1}{72}$	$9\times\frac{1}{72}=\frac{1}{8}$	$\frac{1}{10}\times\frac{1}{8}=\frac{1}{80}$	$10\times\frac{1}{80}=\frac{1}{8}$
$\frac{1}{9}\times\frac{1}{9}=\frac{1}{81}$	$9\times\frac{1}{81}=\frac{1}{9}$	$\frac{1}{10}\times\frac{1}{9}=\frac{1}{90}$	$10\times\frac{1}{90}=\frac{1}{9}$
$\frac{1}{9}\times\frac{1}{10}=\frac{1}{90}$	$9\times\frac{1}{90}=\frac{1}{10}$	$\frac{1}{10}\times\frac{1}{10}=\frac{1}{100}$	$10\times\frac{1}{100}=\frac{1}{10}$
$\frac{1}{9}\times\frac{1}{11}=\frac{1}{99}$	$9\times\frac{1}{99}=\frac{1}{11}$	$\frac{1}{10}\times\frac{1}{11}=\frac{1}{110}$	$10\times\frac{1}{110}=\frac{1}{11}$
$\frac{1}{9}\times\frac{1}{12}=\frac{1}{108}$	$9\times\frac{1}{108}=\frac{1}{12}$	$\frac{1}{10}\times\frac{1}{12}=\frac{1}{120}$	$10\times\frac{1}{120}=\frac{1}{12}$

$\frac{1}{11}\times 1=\frac{1}{11}$	$11\times\frac{1}{11}=1$	$\frac{1}{12}\times 1=\frac{1}{12}$	$12\times\frac{1}{12}=1$
$\frac{1}{11}\times\frac{1}{2}=\frac{1}{22}$	$11\times\frac{1}{22}=\frac{1}{2}$	$\frac{1}{12}\times\frac{1}{2}=\frac{1}{24}$	$12\times\frac{1}{24}=\frac{1}{2}$
$\frac{1}{11}\times\frac{1}{3}=\frac{1}{33}$	$11\times\frac{1}{33}=\frac{1}{3}$	$\frac{1}{12}\times\frac{1}{3}=\frac{1}{36}$	$12\times\frac{1}{36}=\frac{1}{3}$
$\frac{1}{11}\times\frac{1}{4}=\frac{1}{44}$	$11\times\frac{1}{44}=\frac{1}{4}$	$\frac{1}{12}\times\frac{1}{4}=\frac{1}{48}$	$12\times\frac{1}{48}=\frac{1}{4}$
$\frac{1}{11}\times\frac{1}{5}=\frac{1}{55}$	$11\times\frac{1}{55}=\frac{1}{5}$	$\frac{1}{12}\times\frac{1}{5}=\frac{1}{60}$	$12\times\frac{1}{60}=\frac{1}{5}$
$\frac{1}{11}\times\frac{1}{6}=\frac{1}{66}$	$11\times\frac{1}{66}=\frac{1}{6}$	$\frac{1}{12}\times\frac{1}{6}=\frac{1}{72}$	$12\times\frac{1}{72}=\frac{1}{6}$
$\frac{1}{11}\times\frac{1}{7}=\frac{1}{77}$	$11\times\frac{1}{77}=\frac{1}{7}$	$\frac{1}{12}\times\frac{1}{7}=\frac{1}{84}$	$12\times\frac{1}{84}=\frac{1}{7}$
$\frac{1}{11}\times\frac{1}{8}=\frac{1}{88}$	$11\times\frac{1}{88}=\frac{1}{8}$	$\frac{1}{12}\times\frac{1}{8}=\frac{1}{96}$	$12\times\frac{1}{96}=\frac{1}{8}$
$\frac{1}{11}\times\frac{1}{9}=\frac{1}{99}$	$11\times\frac{1}{99}=\frac{1}{9}$	$\frac{1}{12}\times\frac{1}{9}=\frac{1}{108}$	$12\times\frac{1}{108}=\frac{1}{9}$
$\frac{1}{11}\times\frac{1}{10}=\frac{1}{110}$	$11\times\frac{1}{110}=\frac{1}{10}$	$\frac{1}{12}\times\frac{1}{10}=\frac{1}{120}$	$12\times\frac{1}{120}=\frac{1}{10}$
$\frac{1}{11}\times\frac{1}{11}=\frac{1}{121}$	$11\times\frac{1}{121}=\frac{1}{11}$	$\frac{1}{12}\times\frac{1}{11}=\frac{1}{132}$	$12\times\frac{1}{132}=\frac{1}{11}$
$\frac{1}{11}\times\frac{1}{12}=\frac{1}{132}$	$11\times\frac{1}{132}=\frac{1}{12}$	$\frac{1}{12}\times\frac{1}{12}=\frac{1}{144}$	$12\times\frac{1}{144}=\frac{1}{12}$

readily performed without reducing the mixed numbers to improper fractions.

1. Multiply $187\frac{3}{4}$ by $12\frac{2}{3}$.

OPERATION.

$187\frac{3}{4}$
$\underline{\ \ 12\frac{2}{3}}$
$125\frac{1}{6}$ product of $\frac{2}{3}$ times the multiplicand.
$\underline{2253}$ product of 12 times the multiplicand.
$2378\frac{1}{6}$ product of $12\frac{2}{3}$ times the multiplicand.

Multiply the following:

2. 1829 by $12\frac{1}{2}$.
3. 2672 by $18\frac{3}{4}$.
4. $7396\frac{5}{12}$ by $25\frac{3}{5}$.
5. $4563\frac{8}{9}$ by $36\frac{3}{7}$.
6. $8340\frac{7}{8}$ by $126\frac{2}{3}$.
7. $3746\frac{5}{11}$ by $276\frac{8}{9}$.
8. 1728 by $37\frac{5}{12}$.
9. 1694 by $19\frac{1}{4}$.
10. 2736 by $37\frac{3}{4}$.
11. 5937 by $256\frac{5}{6}$.

NOTE TO THE TEACHER.—Thus far the pupil has solved the examples in fractions without having derived any general rules, or formulas, for the several processes; but, if the subject has been thoroughly taught and the fraction table dwelt upon, he is now prepared to formulate rules.

176. Let the pupil formulate a rule for each of the foregoing processes after the following model.

177. To reduce a compound fraction to a simple fraction.

RULE.—*Multiply the numerators together for a new numerator and the denominators together for a new denominator.*

178. Illustrate by appropriate diagrams the following principles.

PRIN. 1.—Multiplying the numerator by any integer multiplies the value of the fraction by that number.

PRIN. 2. — Dividing the numerator by any integer divides the value of the fraction by that number.

HINT. — The number of the parts is decreased while the size of the parts remains unchanged.

PRIN. 3. — Multiplying the denominator by any integer divides the value of the fraction by that number.

PRIN. 4. — Dividing the denominator by any integer multiplies the value of the fraction by that number.

PRIN. 5. — Multiplying both numerator and denominator by the same number does not alter the value of the fraction.

HINT. — Show that the number of parts is increased in the same ratio as their size is decreased.

PRIN. 6. — Dividing both numerator and denominator by the same number does not alter the value of the fraction.

HINT. — Show that the number of parts is decreased in the same ratio as their size is increased.

179. From the provisional definition, "a fraction is one or more of the equal parts of anything," let the pupil be led to the truer definition:

A fraction is one or more of the equal parts of a unit.

A unit is abstract, always; it is one time, or once, a standard of measure.

180. Examples for Oral Solution.

1. What is meant by one-half, one-third, one-fourth?
2. How many halves in a whole? How many thirds? How many fourths? How many fifths?
3. If an apple is divided into two equal parts, what is each part called?
4. How many halves in 1? How many in 2? in 3? in 4? in 5? in 6? in 7?

5. How many thirds in 1? How many in 2? in 3? in 4? in 5? in 6?

6. How many quarters in 1? in 2? in 3? in 4?

7. How many fourths in 3 and 3 fourths?

8. How many fifths in 6 and 4 fifths?

9. How many sixths in 7 and 5 sixths?

10. How many sevenths in 8 and 6 sevenths?

11. How many eighths in 9 and 5 eighths?

12. How many ninths in 12 and 7 ninths?

13. How many wholes in 12 halves?

Analysis.—Since it takes 2 halves to make 1 whole, there are one-half as many wholes as there are halves. One-half of 12 is 6. Therefore in 12 halves there are 6 wholes.

14. How many wholes in $\frac{16}{4}$? in $\frac{15}{3}$? in $\frac{25}{5}$? in $\frac{30}{6}$? in $\frac{42}{7}$? in $\frac{48}{8}$? in $\frac{72}{9}$? in $\frac{96}{12}$? in $\frac{60}{10}$?

15. What is $\frac{1}{3}$ of 12? of 6? of 18? of 24? of 42?

16. What is $\frac{1}{4}$ of 4? of 8? of 12? of 16? of 24? of 48? of 60?

17. What is $\frac{1}{5}$ of 5? of 30? of 45? of 50? of 60? of 40?

18. What is $\frac{1}{6}$ of 6? of 12? of 24? of 48? of 54? of 60?

19. What is $\frac{1}{7}$ of 14? of 28? of 42? of 63? of 84?

20. What is $\frac{1}{8}$ of 16? of 24? of 48? of 64? of 72?

21. If a man earns 60 dollars in 1 month, how many will he earn in $\frac{1}{4}$ of a month? in $\frac{3}{4}$? in $\frac{5}{6}$? in $\frac{7}{12}$?

22. What will $\frac{4}{5}$ of 15 doz. eggs cost at 8 cents a dozen?

23. If Ennis has 120 cents, and Grantland $\frac{7}{12}$ as many, less 10 cents, how many cents has Grantland?

24. A man bought a horse for 108 dollars; he gave $\frac{7}{12}$ as much for a carriage as he paid for the horse, and $\frac{5}{7}$ as much for a harness as he paid for the carriage. How much did all cost him?

25. 6 is $\frac{1}{4}$ of what number?

ANALYSIS. — 6 is $\frac{1}{4}$ of 4 times 6, which is 24.

26. 5 is $\frac{1}{8}$ of what number?

27. 8 is $\frac{1}{9}$ of what number?

28. 12 is $\frac{1}{5}$ of what number?

29. 16 is $\frac{1}{4}$ of what number?

30. 20 is $\frac{1}{3}$ of what number?

31. 25 is how many times 5? $2\frac{1}{2}$? 10?

32. 36 is how many times 6? 3? $1\frac{1}{2}$?

33. 64 is how many times 8? 4? 2? 16? 32?

34. 81 is how many times 9? 3? 27?

35. 45 is how many times 5? 9? 15? 3?

36. 75 is how many times 25? 15? 5? 3?

37. $\frac{3}{4}$ of 20 is how many times 5?

38. $\frac{3}{5}$ of 60 is how many times 9?

39. If $\frac{1}{3}$ of a yard of cloth costs 10 cents, what will a yard cost?

40. If $\frac{1}{5}$ of a pound of coffee cost 6 cents, what will a pound come to?

41. A man gave 25 cents for his dinner, which was $\frac{1}{10}$ of all the money he had. How much money had he?

42. If $\frac{1}{5}$ of a ton of hay cost 3 dollars, what will $\frac{3}{5}$ of a ton cost? 1 ton?

43. A farmer sold some sheep, and gained by the sale 24 dollars which was $\frac{1}{4}$ of what the sheep cost him. What was the cost of the sheep?

44. Frank had 60 apples, and divided $\frac{1}{4}$ of them equally among 3 of his playmates. How many did each receive?

45. A planter sowed 48 acres in oats; $\frac{5}{6}$ of this was just $\frac{1}{3}$ of the number of acres planted in wheat. How many acres did he plant in wheat?

46. Bought a horse and buggy; for the horse I gave 75 dollars, which was $\frac{1}{2}$ of what the buggy cost me. What was the cost of both?

47. If $\frac{3}{4}$ of a man's wages for 1 month amount to 27 dollars, what will his month's wages come to?

48. If $\frac{3}{4}$ of a yard of cloth cost 9 cents, what will be the cost of $\frac{5}{6}$ of a yard?

49. What will a barrel of flour cost, if $\frac{5}{6}$ of it cost 10 dollars?

50. A watch was sold for 12 dollars, which was $\frac{6}{5}$ of what it cost. What did it cost?

51. John paid $\frac{7}{9}$ of his money for a gun; he then gave 4 dollars for ammunition, which was $\frac{2}{3}$ of the money he had left. How much money had he at first?

In $\frac{4}{2}$ how many 1's?
In $\frac{16}{4}$ how many 1's?
In $\frac{25}{5}$ how many 1's?
In $\frac{42}{7}$ how many 1's?
In $\frac{32}{8}$ how many 1's?
In $\frac{24}{3}$ how many 1's?
In $\frac{12}{4}$ how many 1's?
In $\frac{72}{24}$ how many 1's?
In $\frac{51}{17}$ how many 1's?
In $\frac{96}{24}$ how many 1's?
In $\frac{36}{10}$ how many 1's?
In $\frac{7}{2}$ how many 1's?

In $\frac{36}{9}$ how many 1's?
In $\frac{72}{12}$ how many 1's?
In $\frac{84}{12}$ how many 1's?
In $\frac{100}{5}$ how many 1's?
In $\frac{49}{7}$ how many 1's?
In $\frac{36}{12}$ how many 1's?
In $\frac{90}{15}$ how many 1's?
In $\frac{1}{2}$ how many 4ths?
In $\frac{1}{5}$ how many 10ths?
In $\frac{1}{6}$ how many 36ths?
In $\frac{3}{4}$ how many 12ths?
In $\frac{5}{6}$ how many 24ths?

In $\frac{42}{8}$ how many 1's?
In $\frac{50}{6}$ how many 1's?
In $\frac{24}{6}$ how many 2's?
In $\frac{36}{4}$ how many 3's?
In $\frac{64}{8}$ how many 4's?
In $\frac{7}{8}$ how many 48ths?
In $\frac{9}{24}$ how many 72ds?
In $\frac{14}{15}$ how many 90ths?
In $\frac{5}{9}$ how many 54ths?
In $\frac{15}{16}$ how many 64ths?

How many 8ths in $\frac{6}{24}$? in $\frac{24}{32}$? in $\frac{56}{64}$?
How many 6ths in $\frac{8}{12}$? in $\frac{15}{18}$? in $\frac{20}{24}$?
How many 7ths in $\frac{12}{14}$? in $\frac{18}{21}$? in $\frac{36}{42}$?
How many 9ths in $\frac{21}{27}$? in $\frac{32}{36}$? in $\frac{48}{54}$?
How many 5ths in $\frac{12}{15}$? in $\frac{16}{20}$? in $\frac{15}{25}$?
How many 4ths in $\frac{9}{12}$? in $\frac{12}{16}$? in $\frac{30}{40}$?
How many 3ds in $\frac{22}{33}$? in $\frac{28}{42}$? in $\frac{48}{72}$?
How many 12ths in $\frac{30}{36}$? in $\frac{50}{60}$? in $\frac{54}{72}$?
How many 13ths in $\frac{24}{26}$? in $\frac{27}{39}$? in $\frac{48}{52}$?
How many 15ths in $\frac{48}{60}$? in $\frac{72}{90}$? in $\frac{27}{45}$?
How many 20ths in $\frac{90}{100}$? in $\frac{38}{40}$? in $\frac{54}{60}$?
How many 25ths in $\frac{46}{50}$? in $\frac{96}{100}$? in $\frac{42}{150}$?
What is $\frac{4}{8}$ in its lowest terms? $\frac{3}{6}$? $\frac{5}{10}$?
What is $\frac{6}{12}$ in its lowest terms? $\frac{8}{16}$? $\frac{9}{18}$?
What is $\frac{7}{14}$ in its lowest terms? $\frac{10}{20}$? $\frac{12}{24}$?
What is $\frac{7}{21}$ in its lowest terms? $\frac{4}{12}$? $\frac{6}{18}$?
What is $\frac{8}{12}$ in its lowest terms? $\frac{9}{12}$? $\frac{10}{12}$?
What is $\frac{21}{24}$ in its lowest terms? $\frac{20}{24}$? $\frac{18}{24}$?
What is $\frac{75}{100}$ in its lowest terms? $\frac{50}{60}$? $\frac{48}{72}$?
What is $\frac{45}{90}$ in its lowest terms? $\frac{60}{90}$? $\frac{72}{84}$?
What is $\frac{48}{54}$ in its lowest terms? $\frac{36}{48}$? $\frac{30}{36}$?

What is $\frac{27}{54}$ in its lowest terms? $\frac{75}{100}$? $\frac{36}{72}$?

$\frac{4}{5}$ of 30 is how many thirds of 12?

Analysis. — $\frac{4}{5}$ of $30 = 24$; $\frac{1}{3}$ of $12 = 4$; $24 \div 4 = 6$.

$\frac{7}{8}$ of 48 is how many halves of 14?

$\frac{2}{3}$ of 12 is how many sixths of 24?

$\frac{5}{9}$ of 54 is how many fifths of 25?

$\frac{6}{7}$ of 42 is how many thirds of 36?

$\frac{5}{6}$ of 60 is how many eighths of 40?

$\frac{7}{12}$ of 108 is how many sixths of 54?

$\frac{14}{15}$ of 45 is how many thirds of 21?

$\frac{21}{25}$ of 75 is how many thirds of 27?

181. FRACTIONS — PROPER.

A fraction is one or more times the reciprocal of an integer.

Let a quantity measured in terms of a given unit be represented by 5; then its reciprocal (the unit measured in terms of the quantity) will be $\frac{1}{5}$; twice the reciprocal $= \frac{2}{5}$; 3 times $\frac{1}{5} = \frac{3}{5}$; etc.

The reciprocal is $\frac{1}{5}$ the quantity represented by 5 ($\frac{1}{5}$ of 5 or $\frac{5}{5}$), and not its reduced equivalent 1. When thus reduced it loses its fractional character and becomes properly an integer of a lower order. For example, $\frac{3}{4}$ times (of) a gallon is strictly a fraction; but 3 quarts is an integer.

In the first place the fractional idea is explicit; in the second, implicit.

In general, any number thought explicitly in terms of a higher unit (unit of the fraction) is a fraction; thought explicitly in terms of its own unit (fractional unit) is an integer.

To Teacher. — The concrete method of presenting fractions offers an easy and natural introduction to fractions; but it must not be

lost sight of that abstract ideas refuse to be perfectly "concreted." The newsboy "handles" coppers, nickels, dimes, quarters, halves, and dollars with as little difficulty as the grocer measures syrup or vinegar by the gill, pint, quart, gallon. But this facility in dealing with fractions in the concrete does not, however, necessarily imply a mastery of those processes that are adequately presented only in the abstract.

182. From the conception of division embodied in its definition, it follows that a fraction is an indicated division. The numerator is the dividend and the denominator the divisor.

183. The value of a fraction is the quotient of the numerator divided by the denominator.

184. Every fractional process has its rationale in some one of the following principles:

PRIN. 1.—Multiplying the numerator by any integer multiplies the value of the fraction by that number; as,

$$\frac{48}{12} = 4; \ \frac{48 \times 2}{12} = 2 \times 4 = 8.$$

PRIN. 2.—Dividing the numerator by any integer divides the value of the fraction by that number; as,

$$\frac{48}{12} = 4; \ \frac{48 \div 2}{12} = 4 \div 2 = 2.$$

PRIN. 3.—Multiplying the denominator by any integer divides the value of the fraction by that number; as,

$$\frac{48}{12} = 4; \ \frac{48}{12 \times 2} = 4 \div 2 = 2.$$

PRIN. 4.—Dividing the denominator by any integer multiplies the value of the fraction by that number; as,

$$\frac{48}{12} = 4; \ \frac{48}{12 \div 2} = 4 \times 2 = 8.$$

Prin. 5. — Multiplying both numerator and denominator by the same number does not alter the value of the fraction; as,

$$\frac{48}{12}=4;\ \frac{48\times 2}{12\times 2}=4.$$

Prin. 6. — Dividing both numerator and denominator by the same number does not alter the value of the fraction; as,

$$\frac{48}{12}=4;\ \frac{48\div 2}{12\div 2}=4.$$

Prin. 7. — Or generally, the value of a fraction varies directly as the numerator, and reciprocally as the denominator.

Prin. 8. — The sum of the quotients equals the quotient of the sum; as,

$$\frac{8}{2}(=4)+\frac{4}{2}(=2)=\frac{8+4}{2}=6.$$

Prin. 9. — The difference of the quotients equals the quotient of the difference; as,

$$\frac{12}{2}(=6)-\frac{4}{2}(=2)=\frac{12-4}{2}=\frac{8}{2}=4.$$

185. Reduction of Fractions.

The reduction of a fraction is the process of changing its terms without altering its value.

186. *Case I.*

To reduce a fraction to its lowest terms.

A fraction is in its lowest terms when its numerator and denominator are prime to each other.

1. Reduce $\frac{9}{12}$ to its lowest terms.

OPERATION.

$$\frac{9 \div 3}{12 \div 3} = \frac{3}{4}.$$

Or,

$$\frac{9}{12} = \frac{3 \times \cancel{3}}{4 \times \cancel{3}} = \frac{3}{4}.$$

EXPLANATION. — Divide both terms by their common factor 3 (Prin. 6). Since 3 is their only common factor, or their G. C. D., its removal leaves the terms prime to each other. Hence the fraction has been reduced to its lowest terms.

RULE. — *Divide both terms of the fraction by any common divisor and continue the division until no number greater than* 1 *will exactly divide them.*

Or,

Divide both terms by their greatest common divisor.

Reduce to their lowest terms:

2. $\frac{85}{95}$; $\frac{72}{108}$; $\frac{119}{121}$; $\frac{12}{18}$; $\frac{24}{30}$; $\frac{42}{56}$; $\frac{48}{72}$.

3. $\frac{14}{21}$; $\frac{21}{28}$; $\frac{16}{20}$; $\frac{20}{48}$; $\frac{16}{28}$; $\frac{12}{16}$; $\frac{52}{78}$.

4. $\frac{88}{99}$; $\frac{47}{63}$; $\frac{27}{93}$; $\frac{234}{468}$; $\frac{128}{324}$; $\frac{814}{962}$; $\frac{420}{504}$.

5. $\frac{394}{672}$; $\frac{272}{425}$; $\frac{126}{210}$; $\frac{108}{132}$; $\frac{168}{336}$; $\frac{132}{396}$; $\frac{112}{168}$.

6. $\frac{135}{225}$; $\frac{232}{493}$; $\frac{210}{350}$; $\frac{352}{384}$; $\frac{216}{360}$; $\frac{126}{189}$; $\frac{126}{210}$.

7. $\frac{294}{462}$; $\frac{126}{162}$; $\frac{132}{154}$; $\frac{125}{175}$; $\frac{252}{216}$; $\frac{192}{224}$.

8. $\frac{135}{162}$; $\frac{175}{245}$; $\frac{215}{252}$; $\frac{147}{210}$; $\frac{154}{198}$; $\frac{252}{324}$.

9. $\frac{245}{525}$; $\frac{301}{480}$; $\frac{392}{672}$; $\frac{252}{624}$; $\frac{378}{648}$.

187. *Case II.*

To reduce a fraction to higher terms.

A fraction is expressed in higher terms when an integer has been introduced as a factor into both its numerator and denominator.

1. Reduce $\frac{3}{4}$ to 12ths.

OPERATION.

$$12 \div 4 = 3;$$
$$\frac{3 \times 3}{4 \times 3} = \frac{9}{12}.$$

EXPLANATION. — Multiply both terms (Prin. 5) by the number that will change the denominator 4 to 12, which is $12 \div 4 = 3$. 3 introduced as a factor into both terms brings the fraction to higher terms.

RULE. — *Multiply both terms of the fraction by that number that will bring the given denominator to the required denominator.*

Reduce the following:

2. $\frac{17}{19}$ to 95ths.
3. $\frac{2}{3}$ to 21sts.
4. $\frac{8}{9}$ to 99ths.
5. $\frac{5}{9}$ to 27ths.
6. $\frac{2}{3}$ to 108ths.
7. $\frac{3}{4}$ to 28ths.
8. $\frac{7}{9}$ to 63ds.
9. $\frac{4}{15}$ to 45ths.
10. $\frac{4}{5}$ to 30ths.
11. $\frac{4}{5}$ to 20ths.
12. $\frac{3}{11}$ to 99ths.
13. $\frac{8}{18}$ to 72ds.
14. $\frac{3}{4}$ to 56ths.
15. $\frac{5}{12}$ to 48ths.
16. $\frac{13}{70}$ to 210ths.
17. $\frac{17}{24}$ to 360ths.
18. $\frac{19}{57}$ to 798ths.
19. $\frac{5}{1}$ to 20ths.
20. $\frac{18}{1}$ to 72ds.
21. $\frac{21}{1}$ to 83ds.

188. *Case III.*

To reduce a mixed number to an improper fraction.

1. Reduce $25\frac{3}{4}$ to an improper fraction.

OPERATION.

$$25 = \frac{25}{1};\ \frac{25 \times 4}{1 \times 4} = \frac{100}{4};$$
$$\frac{100}{4} + \frac{3}{4} = \frac{103}{4}.$$

EXPLANATION. — Reduce the integral part to a fraction having the denominator of the fractional part (Case II.) and add the integer thus reduced to the fraction (Prin. 8). Whence the required improper fraction is $\frac{103}{4}$.

Hence the

RULE. — *Multiply the integer by the denominator, add the numerator to the product, and set the sum over the denominator.*

Reduce the following to improper fractions:

2. $12\frac{1}{2}$. 7. $41\frac{5}{7}$. 12. $39\frac{5}{11}$. 17. $99\frac{5}{11}$.
3. $75\frac{3}{4}$. 8. $60\frac{2}{5}$. 13. $43\frac{13}{15}$. 18. $49\frac{5}{9}$.
4. $18\frac{1}{8}$. 9. $72\frac{5}{9}$. 14. $79\frac{11}{14}$. 19. $127\frac{15}{17}$.
5. $19\frac{7}{8}$. 10. $83\frac{3}{7}$. 15. $63\frac{17}{19}$. 20. $345\frac{27}{31}$.
6. $37\frac{3}{5}$. 11. $27\frac{2}{5}$. 16. $84\frac{21}{25}$. 21. $943\frac{81}{121}$.
22. $845\frac{31}{188}$. 23. $793\frac{311}{571}$.

189. *Case IV.*

To reduce an improper fraction to a whole or mixed number.

1. Reduce $\frac{64}{9}$ to a mixed number.

OPERATION.

$$\tfrac{64}{9} = 64 \div 9 = 7\tfrac{1}{9}.$$

Or,

$$\frac{64}{9} = \frac{64 \div 9}{9 \div 9} = \frac{64 \div 9}{1} = 7\tfrac{1}{9}.$$

EXPLANATION. — Perform the indicated division; or divide both terms by the denominator (Prin. 6).

RULE. — *Divide the numerator by the denominator.*

Reduce the following to whole or mixed numbers:

2. $\frac{108}{12}$. 5. $\frac{1088}{48}$. 8. $\frac{8567}{42}$. 11. $\frac{9548}{278}$. 14. $\frac{189648}{158}$.
3. $\frac{1728}{144}$. 6. $\frac{1956}{112}$. 9. $\frac{6278}{327}$. 12. $\frac{89734}{1024}$. 15. $\frac{864205}{765}$.
4. $\frac{249}{39}$. 7. $\frac{8596}{154}$. 10. $\frac{8972}{558}$. 13. $\frac{78510}{225}$.

190. *Case V.*

To reduce two or more fractions to similar fractions; that is, to others having a common denominator.

Fractions are said to have a common denominator when they have the same denominator. The common denominator must be a multiple, better the least common multiple, of the several denominators.

1. Reduce $\frac{3}{4}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$ to a common denominator.

OPERATION.

$$24 \div 4 = 6; \quad \frac{3 \times 6}{4 \times 6} = \frac{18}{24}.$$

$$24 \div 3 = 8; \quad \frac{2 \times 8}{3 \times 8} = \frac{16}{24}.$$

$$24 \div 6 = 4; \quad \frac{5 \times 4}{6 \times 4} = \frac{20}{24}.$$

$$24 \div 8 = 3; \quad \frac{7 \times 3}{8 \times 3} = \frac{21}{24}.$$

EXPLANATION. — Find the L. C. M. of the denominators to be 24.

Reduce the several fractions to others having 24 for their common denominator, by Case II. The fractions thus found are $\frac{18}{24}$, $\frac{16}{24}$, $\frac{20}{24}$, and $\frac{21}{24}$.

RULE. — *Find the L. C. M. of the denominators for the common denominator, and multiply both terms of each fraction by that number that will severally bring the given denominators to the required denominator.*

Reduce the following to similar fractions:

2. $\frac{5}{7}$; $\frac{3}{8}$; $\frac{11}{12}$; $\frac{3}{4}$; $\frac{13}{14}$.
3. $\frac{15}{16}$; $\frac{8}{21}$; $\frac{5}{14}$; $\frac{6}{7}$; $\frac{7}{12}$.
4. $\frac{1}{2}$; $\frac{3}{8}$; $\frac{5}{6}$; $\frac{7}{16}$; $\frac{33}{64}$.
5. $\frac{1}{7}$; $\frac{3}{14}$; $\frac{12}{21}$; $\frac{29}{42}$.
6. $\frac{2}{3}$; $\frac{3}{4}$; $\frac{5}{7}$; $\frac{8}{21}$; $\frac{15}{42}$.
7. $\frac{1}{5}$; $\frac{7}{8}$; $\frac{9}{16}$; $\frac{13}{20}$; $\frac{17}{40}$.
8. $\frac{2}{9}$; $\frac{3}{14}$; $\frac{13}{28}$; $\frac{17}{42}$.
9. $\frac{2}{3}$; $\frac{8}{18}$; $\frac{11}{39}$; $\frac{17}{78}$.
10. $\frac{2}{3}$; $\frac{3}{5}$; $\frac{7}{15}$; $\frac{8}{25}$.
11. $\frac{1}{5}$; $\frac{3}{8}$; $\frac{7}{16}$; $\frac{8}{20}$; $\frac{14}{40}$.
12. $\frac{11}{16}$; $\frac{13}{20}$; $\frac{3}{4}$; $\frac{4}{15}$; $\frac{1}{2}$.
13. $\frac{7}{9}$; $\frac{7}{12}$; $\frac{7}{15}$; $\frac{7}{20}$.
14. $\frac{2}{21}$; $\frac{6}{210}$; $\frac{13}{42}$; $\frac{61}{84}$; $\frac{8}{10}$.
15. $\frac{7}{9}$; $\frac{7}{12}$; $\frac{13}{18}$; $\frac{23}{24}$; $\frac{5}{36}$.
16. $\frac{5}{9}$; $\frac{13}{35}$; $\frac{23}{45}$; $\frac{5}{63}$; $\frac{91}{115}$.
17. $\frac{6}{7}$; $\frac{9}{14}$; $\frac{5}{12}$; $\frac{15}{28}$; $\frac{5}{6}$.
18. $\frac{3}{8}$; $\frac{3}{4}$; $\frac{5}{16}$; $\frac{21}{64}$; $\frac{11}{256}$.
19. $\frac{11}{12}$; $\frac{5}{9}$; $\frac{7}{15}$; $\frac{4}{5}$; $\frac{2}{3}$.
20. $\frac{6}{7}$; $\frac{5}{12}$; $\frac{13}{15}$; $\frac{11}{27}$; $\frac{21}{35}$.
21. $\frac{14}{75}$; $\frac{13}{75}$; $\frac{20}{21}$; $\frac{15}{56}$; $\frac{17}{35}$.

191. ADDITION AND SUBTRACTION.

Similar fractions only can be added or subtracted.

Find the sum of $\frac{2}{7}$, $\frac{1}{3}$, $\frac{3}{14}$.

OPERATION.

$$\tfrac{2}{7}+\tfrac{1}{3}+\tfrac{3}{14}=\tfrac{12}{42}+\tfrac{14}{42}+\tfrac{9}{42};$$

$$\frac{12}{42}+\frac{14}{42}+\frac{9}{42}=\frac{12+14+9}{42}=\frac{35}{42}=\frac{5}{6}.$$

EXPLANATION. — Reduce to similar fractions (Case V.). Add the numerators (Prin. 8). Whence the sum is $\frac{35}{42}=\frac{5}{6}$.

Find the difference between $\frac{5}{6}$ and $\frac{3}{4}$.

OPERATION.

$$\tfrac{5}{6}-\tfrac{3}{4}=\tfrac{10}{12}-\tfrac{9}{12};$$

$$\frac{10}{12}-\frac{9}{12}=\frac{10-9}{12}=\frac{1}{12}.$$

EXPLANATION. — Reduce to similar fractions (Case V.). Subtract the numerator of the subtrahend from that of the minuend (Prin. 9). Whence the difference is $\frac{1}{12}$.

RULE FOR ADDITION. — *Reduce the fractions to similar fractions when necessary; then add the numerators and place the sum over the common denominator.*

RULE FOR SUBTRACTION. — *Reduce the fractions to similar fractions when necessary; then subtract the numerator of the subtrahend from the numerator of the minuend and place the difference thus found over the common denominator.*

Perform the indicated operations upon the following:

1. $\frac{7}{15}+\frac{13}{16}$; $\frac{5}{14}+\frac{17}{21}+\frac{13}{45}$; $\frac{27}{28}+\frac{9}{14}+\frac{8}{35}$.
2. $\frac{21}{26}+\frac{37}{56}+\frac{97}{189}$; $\frac{13}{14}+\frac{7}{18}+\frac{23}{24}$; $\frac{15}{16}-\frac{3}{5}$.
3. $\frac{9}{18}+\frac{47}{52}+\frac{23}{78}$; $\frac{15}{39}+\frac{17}{52}+\frac{19}{78}$; $\frac{17}{21}-\frac{3}{7}$.
4. $\frac{101}{135}+\frac{89}{180}$; $\frac{73}{126}+\frac{61}{140}+\frac{133}{154}$; $7-\frac{5}{8}$.
5. $\frac{72}{117}+\frac{83}{312}$; $\frac{97}{182}+\frac{79}{144}+\frac{123}{288}+\frac{217}{324}$.
6. $\frac{219}{315}+\frac{321}{420}$; $\frac{113}{448}+\frac{333}{512}$; $\frac{213}{432}-\frac{317}{936}$.
7. $\frac{1913}{2310}+\frac{2173}{3150}$; $\frac{589}{720}-\frac{853}{1736}$; $17-\frac{15}{16}$.

192. To add and subtract mixed numbers.

1. Find the sum of $7\frac{3}{4}$, $5\frac{2}{3}$, $18\frac{5}{6}$.

OPERATION.

$$
\begin{aligned}
7 + 5 + 18 &= 30\\
\tfrac{3}{4} + \tfrac{2}{3} + \tfrac{5}{6} &= \underline{2\tfrac{1}{4}}\\
&\ 32\tfrac{1}{4}
\end{aligned}
$$

Explanation. — Find the sum of the integers, which is 30, and the sum of the fractions, which is $2\frac{1}{4}$. Then the sum of both integers and fractions will be $30 + 2\frac{1}{4} = 32\frac{1}{4}$.

Rule. — *Add the integers and fractions separately; then add their sums.*

2. From $65\frac{2}{3}$ take $23\frac{5}{6}$.

OPERATION.

$$
\begin{aligned}
65\tfrac{2}{3} &= 65\tfrac{8}{12}\\
23\tfrac{5}{6} &= \underline{23\tfrac{10}{12}}\\
&\ 41\tfrac{10}{12}(\tfrac{5}{6})
\end{aligned}
$$

Explanation. — Reduce the fractional parts to similar fractions. The fraction of the subtrahend is larger than that of the minuend. Therefore "borrow" 1 unit ($= \frac{12}{12}$) from the units' place and add it to the fraction of the minuend, $\frac{8}{12}$; $\frac{8}{12} + \frac{12}{12} = \frac{20}{12}$; $\frac{20}{12} - \frac{10}{12} = \frac{10}{12}$ or $\frac{5}{6}$. Set down the fraction to the right of the units' place and proceed, 3 from 4, etc., as in subtraction of integers.

3. $425\frac{5}{7} + 671\frac{7}{8}$; $325\frac{5}{14} + 136\frac{6}{7}$; $129\frac{13}{24} - 37\frac{14}{15}$.

4. $227\frac{5}{9} + 183\frac{7}{27}$; $673\frac{3}{7} + 972\frac{7}{9}$; $329\frac{6}{7} - 225\frac{7}{9}$.

5. $864\frac{3}{7} + 460\frac{13}{21}$; $796\frac{16}{21} + 549\frac{15}{16}$; $249\frac{6}{13} - 87\frac{5}{11}$.

193. Multiplication and division.

To multiply a fraction by an integer.

1. Multiply $\frac{7}{12}$ by 2.

OPERATION.

$\frac{7}{12} \times 2 = \frac{14}{12} = 1\frac{1}{6}$.

Or,

$\frac{7}{12} \times 2 = \frac{7}{6} = 1\frac{1}{6}$.

Explanation. — Multiply the numerator by the integer (Prin. 1); or divide the denominator by the integer (Prin. 4). Hence the result, $\frac{14}{12}$ or $\frac{7}{6} = 1\frac{1}{6}$.

Multiply the following:

2. $\frac{15}{16} \times 8$.
3. $\frac{13}{24} \times 6$.
4. $\frac{17}{21} \times 7$.
5. $\frac{13}{18} \times 3$.
6. $\frac{11}{18} \times 9$.
7. $\frac{23}{32} \times 16$.
8. $\frac{35}{72} \times 24$.
9. $\frac{21}{32} \times 5$.
10. $\frac{45}{88} \times 10$.
11. $\frac{95}{108} \times 12$.
12. $\frac{65}{1728} \times 144$.
13. $\frac{73}{90} \times 18$.
14. $\frac{37}{48} \times 24$.
15. $\frac{31}{42} \times 21$.
16. $\frac{99}{108} \times 54$.

194. To divide a fraction by an integer.

1. Divide $\frac{10}{12}$ by 5.

FIRST OPERATION.

$$\frac{10}{12} \div 5 = \frac{10 \div 5}{12} = \frac{2}{12}\left(\frac{1}{6}\right).$$

SECOND OPERATION.

$$\frac{10}{12} \div 5 = \frac{10}{12 \times 5} = \frac{10}{60}\left(\frac{1}{6}\right).$$

THIRD OPERATION.

$$\tfrac{10}{12} \div 5 = \tfrac{1}{5} \text{ of } \tfrac{10}{12} = \tfrac{2}{12}\ (\tfrac{1}{6}).$$

FOURTH OPERATION.

$$\tfrac{10}{12} \div 5 = \tfrac{1}{5} \text{ of } \tfrac{10}{12} = \tfrac{10}{60}\ (\tfrac{1}{6}).$$

EXPLANATION.—Divide the numerator or multiply the denominator by the integer (Prins. 2 and 3) as in the first and second operations. Or, since by the definition of division to divide a quantity by 5 is to take it as many times as there are units in its reciprocal $\frac{1}{5}$, take $\frac{10}{12}$ $\frac{1}{5}$ of a time, setting down the step in the form of a compound fraction.

RULE. —*Divide the numerator, when it can be exactly done, or multiply the denominator by the integer.*

Or,

Write the division in the form of the compound fraction, and multiply the numerators together for a new numerator, and the denominators together for a new denominator.

Divide the following:

2. $\frac{44}{5}$ by 2.
3. $\frac{8}{9}$ by 4.
4. $\frac{9}{10}$ by 3.
5. $\frac{15}{16}$ by 5.
6. $\frac{18}{21}$ by 6.
7. $\frac{39}{42}$ by 13.
8. $\frac{99}{100}$ by 33.
9. $\frac{72}{91}$ by 24.
10. $\frac{81}{98}$ by 27.
11. $\frac{54}{61}$ by 27.
12. $\frac{1250}{1863}$ by 25.
13. $\frac{64}{72}$ by 8.
14. $\frac{1728}{2165}$ by 144.
15. $\frac{625}{731}$ by 25.
16. $\frac{1296}{1898}$ by 36.
17. $\frac{784}{985}$ by 28.
18. $\frac{5329}{6931}$ by 73.
19. $\frac{5625}{8241}$ by 75.

195. To multiply a fraction by a fraction.

1. Multiply $\frac{3}{4}$ by $\frac{2}{5}$.

OPERATION.

$\frac{3}{4} \times \frac{2}{5} = \frac{2}{5}$ of $\frac{3}{4} = \frac{6}{20}\ (\frac{3}{10})$.

EXPLANATION. — To multiply by $\frac{2}{5}$ is to take the quantity to be multiplied $\frac{2}{5}$ of a time. 1 time $\frac{3}{4}$ is $\frac{3}{4}$; $\frac{2}{5}$ times $\frac{3}{4}$ is $\frac{2}{5}$ of $\frac{3}{4}$; $\frac{2}{5}$ of $\frac{3}{4}$ is a compound fraction.

RULE. — *Write the fractions in the form of the compound fraction and apply the rule for the reduction of compound fractions to simple fractions.*

196. To divide by an integer or a fraction.

NOTE. — The student understands upon what principle the integer and the fractional unit stand related as reciprocals, each of the other; how $\frac{1}{4}$ is the reciprocal of 4, and 4 the reciprocal of $\frac{1}{4}$. What then is the reciprocal of $\frac{3}{4}$? $\frac{3}{4}$ is $3 \times \frac{1}{4}$, and the reciprocal of $3 \times \frac{1}{4}$ is $\frac{1}{3} \times 4$, which is $\frac{4}{3}$.

For convenience, integers may be written in the fractional form with the denominator 1 ($6 = \frac{6}{1}$), and, as a memory help, the reciprocal may be thought of as the fraction inverted.

1. Divide $\frac{5}{12}$ by 2; $\frac{3}{4}$ by $\frac{2}{3}$.

PROCESS.

$\frac{5}{12} \div 2 = \frac{1}{2}$ of $\frac{5}{12} = \frac{5}{24}$.

$\frac{3}{4} \div \frac{2}{3} = \frac{3}{2}$ of $\frac{3}{4} = \frac{9}{8}$

EXPLANATION. — By definition, division is the process of taking one number as many times as there are units in the reciprocal of another. Whence the following

RULE. — *To divide by any number, multiply by its reciprocal.*

Perform the indicated operations upon the following:

2. $\frac{8}{9} \div 3$.
3. $\frac{11}{17} \div \frac{2}{3}$.
4. $6\frac{3}{5} \div \frac{1}{3}$.
5. $\frac{2}{3}$ of $\frac{3}{4} \div \frac{1}{2}$ of $\frac{2}{3}$.
6. $\frac{3}{5} \div \frac{3}{4}$.
7. $\frac{6}{7} \div \frac{3}{8}$.
8. $\frac{5}{7} \div \frac{6}{11}$.
9. $\frac{12}{13} \div \frac{3}{4}$.
10. $\frac{6}{7} \div 3$.
11. $\frac{8}{9} \div 4$.
12. $\frac{15}{16} \div 5$.
13. $\frac{10}{13} \div \frac{2}{3}$.
14. $\frac{21}{23} \div \frac{3}{7}$.
15. $\frac{11}{23} \div \frac{5}{9}$.
16. $\frac{5}{21} \div \frac{3}{5}$.
17. $\frac{8}{17} \div \frac{5}{8}$.
18. $\frac{12}{13} \div \frac{5}{6}$.
19. $5\frac{1}{2} \div \frac{3}{11}$.
20. $7\frac{1}{2} \div 2\frac{1}{2}$.
21. $18\frac{3}{4} \div 12\frac{1}{2}$.

197. Another method of dividing one fraction by another.

1. Divide $\frac{6}{12}$ by $\frac{2}{3}$.

OPERATION.

$$\frac{3}{4}\times\frac{2}{3}=\frac{3\times2}{4\times3}=\frac{6}{12}.$$

Hence,

$$\frac{6}{12}\div\frac{2}{3}=\frac{6\div2}{12\div3}=\frac{3}{4}.$$

Explanation. — Since division is the converse of multiplication, and the multiplication of fractions is performed by multiplying the numerators together for a new numerator, and the denominators together for a new denominator, the division of one fraction by another may be performed by dividing the numerator of the dividend by the numerator of the divisor, and the denominator of the dividend by the denominator of the divisor.

Rule. — *Divide the numerator of the dividend by the numerator of the divisor for the numerator of the quotient, and the denominator of the dividend by the denominator of the divisor for the denominator of the quotient.*

Note. — If the division cannot be exactly done, multiply both terms of the dividend by the L. C. M. of the terms of the divisor.

Divide:

2. $\frac{12}{18}$ by $\frac{2}{3}$.
3. $\frac{9}{27}$ by $\frac{3}{9}$.
4. $\frac{27}{30}$ by $\frac{3}{10}$.
5. $\frac{3}{4}$ by $\frac{2}{3}$.
6. $\frac{2}{3}$ by $\frac{3}{4}$.
7. $7\frac{1}{2}$ by $2\frac{1}{2}$.
8. $5\frac{1}{4}$ by $1\frac{1}{2}$.
9. $18\frac{3}{4}$ by $6\frac{1}{4}$.

198. Examples for Review.

1. If 6 men consume $\frac{9}{10}$ of a barrel of flour in 1 month, how much will 1 man consume in the same time?

2. If a bushel of corn is worth $\frac{3}{4}$ of a dollar, what is $\frac{1}{4}$ of a bushel worth?

3. From $\frac{21}{20}$ take $\frac{20}{21}$.

4. A farmer owning $172\frac{3}{16}$ acres of land purchased $54\frac{1}{18}$ acres at one time and $6\frac{11}{40}$ acres at another time. How much land had he after the last purchase?

5. A wall was built by 4 men. The first worked $7\frac{1}{4}$ days; the second, $8\frac{4}{5}$ days; the third, $9\frac{3}{8}$ days; the fourth, $4\frac{2}{3}$ days. How many days did they all work at it?

6. At $5\frac{1}{3}$ cents a pound, how many pounds of rice can be bought with 48 cents?

7. A man owning $378\frac{2}{3}$ acres of land sold $93\frac{2}{5}$ acres. How many acres had he left?

8. What is the cost of $5\frac{1}{4}$ cords of wood, at $4\frac{2}{3}$ dollars a cord?

9. What is the cost of $6\frac{1}{2}$ gallons of syrup, at $87\frac{1}{2}$ cents a gallon?

10. At $\frac{4}{5}$ of a dollar a hat, how many hats can be bought for $11\frac{1}{5}$ dollars?

11. If $2\frac{3}{5}$ boxes of oranges cost $5\frac{1}{5}$ dollars, what is the cost of 1 box?

12. Express in lowest terms the product of $1 + \frac{1}{9} + \frac{2}{3} + \frac{1}{27}$ by $2 + \frac{1}{2} + \frac{3}{4} + \frac{15}{16}$.

13. What will $6\frac{1}{2}$ gallons of kerosene oil cost at $18\frac{2}{3}$ cents a gallon?

14. How many barrels of apples at $4\frac{1}{2}$ dollars a barrel can be bought for 42 dollars?

15. A farmer, owning 387 acres of land, at one time sold $122\frac{1}{4}$ acres, and at another, $93\frac{4}{5}$ acres. How many acres has he left?

16. What must I pay for $19\frac{1}{3}$ yards of cloth, at $14\frac{1}{2}$ cents a yard?

17. If a man earns $2\frac{2}{5}$ dollars a day, how much will he earn in $5\frac{3}{4}$ days?

18. If a fruit dealer makes $3\frac{1}{8}$ cents on every quart of cherries he sells, how many quarts must he sell to make $87\frac{1}{2}$ cents?

19. What will 16⅔ yards of cloth cost, at 16⅔ cents a yard?

20. If ⅝ of a yard of flannel costs 40 cents, what will 7½ yards cost?

21. From a piece of cloth containing 35⅚ yards, 18¾ yards were cut off. How many yards were left?

22. In a certain orchard ½ of the trees bear peaches, ¼ of them pears, ⅙ of them cherries, 125 of them apples, and 75 of them plums. How many trees are there in the orchard?

23. If John earns ⅞ of a dollar a day, how long will it take him to earn 10 dollars?

24. What is the cost of 15½ tons of coal, at 3⅓ dollars a ton?

25. A spent ½, ¼, and ⅙ of his money, and had 100 dollars left. What had he at first?

26. If I spend ½ of my money, then ⅓ of what is left, then ¼ of what remains, what part of the whole will I have left?

27. What will 8756½ pounds of sugar cost, at 7¼ cents a pound?

28. A man paid 8756½ dollars for sugar, at 15 dollars a barrel. How many barrels did he buy?

29. What is the value of 18½ barrels of apples, at 4⅓ dollars a barrel?

30. A grocer gains 7⅓ cents on each pound of butter he sells. How many pounds must he sell to gain 113⅔ cents?

31. How much must be paid for 23¾ tons of coal, at 5⅔ dollars a ton?

32. A boy, having 27½ dollars, spent 17⅔ dollars. How much money had he left?

33. If a man walks 3 miles in $\frac{2}{3}$ of an hour, in what time does he walk 1 mile?

34. What is the quotient of $99\frac{1}{7}$ divided by $19\frac{4}{5}$?

35. A young man, having 98 dollars in the savings bank, drew out $53\frac{4}{5}$ dollars at one time and $17\frac{2}{5}$ dollars at another. How much had he left in bank?

36. If a man can perform a certain journey in $22\frac{1}{2}$ days, in what time will he perform $\frac{1}{3}$ of the journey?

37. If $\frac{2}{5}$ of a yard of cloth cost $\frac{7}{8}$ of a dollar, what will 1 yard cost?

38. A man bought a farm for $3437\frac{1}{2}$ dollars, paying $12\frac{1}{2}$ dollars an acre. How many acres of land did he buy?

39. The difference between $\frac{1}{4}$ and $\frac{1}{5}$ of a number is $63\frac{1}{2}$. What is the number?

40. If $\frac{5}{6}$ of a gallon of vinegar cost $\frac{5}{8}$ of a dollar, what will $\frac{5}{9}$ of a gallon cost?

41. A paid 1200 dollars for $\frac{1}{2}$ interest in an ice factory. What part had he left after selling $\frac{3}{5}$ of $\frac{4}{9}$ of his share, and what was it worth?

42. The sum of two numbers is $\frac{7}{8}$, and the smaller number is $\frac{5}{24}$. What is the larger number?

43. What must be added to $\frac{3}{7}$ of $\frac{5}{8}$ to make it equal to $\frac{4}{5}$ of $2\frac{1}{3}$?

44. $\frac{3}{4}$ of $\frac{5}{6}$ of what number gives 180?

45. A lady bought $17\frac{1}{2}$ yards of silk, at $1\frac{3}{4}$ dollars a yard; $2\frac{1}{3}$ yards of velvet, at $3\frac{1}{4}$ dollars a yard; 5 yards of lace, at $\frac{7}{8}$ of a dollar a yard; 3 pairs of gloves, at $1\frac{1}{4}$ dollars a pair; 1 dozen handkerchiefs, at $\frac{1}{3}$ of a dollar each. What was the amount of her bill?

46. Into how many pieces of $18\frac{2}{3}$ yards each can $186\frac{2}{3}$ yards of silk be cut?

47. If I buy $\frac{7}{9}$ of $\frac{5}{18}$ of a store, and then sell $\frac{2}{3}$ of what I bought, what part of the store do I own?

48. A bought of B $87\frac{1}{2}$ pounds of coffee, at $16\frac{2}{3}$ cents a pound; and B, at the same time, purchased of A 25 bushels of corn, at $63\frac{7}{8}$ cents a bushel. What is the difference in their bills, and to which one is it due?

49. A man owing the following amounts, \$ $55\frac{1}{7}$, \$ $31\frac{3}{8}$, \$ $27\frac{2}{15}$, \$ $58\frac{3}{17}$, has but \$ 115. After paying what he can, how much will he still owe?

50. At $\frac{5}{6}$ of a dollar a yard, what will $\frac{7}{8}$ of a yard of cloth cost?

51. If I sell a $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{8}$ interest in my business for \$ 4000, what is the business worth?

52. A grain dealer bought $358\frac{2}{3}$ bushels of corn, at $\frac{2}{3}$ of a dollar a bushel; 420 bushels of oats, at $\frac{3}{8}$ of a dollar a bushel; $146\frac{1}{2}$ bushels of rye, at $\frac{5}{8}$ of a dollar a bushel; $651\frac{1}{8}$ bushels of wheat, at $\frac{7}{8}$ of a dollar a bushel. What must he pay for all?

53. If $\frac{3}{8}$ of a yard of cloth be worth $\frac{17}{18}$ of a dollar, what is the value of a yard?

54. What fraction is that to which if $\frac{2}{7}$ of $\frac{5}{9}$ be added the sum will be 1?

55. Simplify $\frac{\frac{2}{3}}{\frac{3}{4}}$.

SOLUTION. $\frac{\frac{2}{3}}{\frac{3}{4}} = \frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$.

By inspection it may be seen that 8 is the product of the extremes, and that 9 is the product of the means. Whence, to reduce a complex fraction to a simple one, —

Multiply the extremes together for the numerator, and the means together for the denominator.

56. Simplify $\frac{4}{\frac{1}{3}}\left(=\frac{\frac{4}{1}}{\frac{1}{3}}\right)$, $\frac{2\frac{1}{2}}{5}$, $\frac{\frac{6}{8}}{\frac{5}{9}}$, $\frac{4}{\frac{3}{5}}$, $\frac{\frac{2}{3}\text{ of }\frac{3}{4}+\frac{1}{2}\times\frac{4}{5}}{9-\frac{1}{3}\times 6\div 2}$.

Note. — The signs of multiplication and division take precedence over the signs of addition and subtraction, because an indicated product (or quotient) is the true product (or quotient) (Art. 51), that is, the plus and minus signs enclose the *terms* of an expression.

Whence $\frac{\frac{2}{3}\text{ of }\frac{3}{4}+\frac{1}{2}\times\frac{4}{5}}{9-\frac{1}{3}\times 6\div 2}=\frac{(\frac{2}{3}\text{ of }\frac{3}{4})+(\frac{1}{2}\times\frac{4}{5})}{9-(\frac{1}{3}\times 6\div 2)}$.*

57. Simplify $\frac{\frac{6}{8}+5\frac{1}{3}+\frac{2}{3}\text{ of }4\frac{1}{2}}{5\frac{3}{4}-3\frac{2}{3}+\frac{1}{2}\text{ of }\frac{4}{5}}$.

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* $\frac{1}{3}\times 6\div 2=\frac{1\times 6}{3}\div 2$.

CHAPTER VIII.

DECIMAL FRACTIONS, OR DECIMALS.

199. The reciprocals of the integral orders of units above the first — tens, hundreds, thousands, etc. — give rise to corresponding fractional orders of units — tenths, hundredths, thousandths, etc. — known as **Decimals.**

200. These decimal orders sustain to each other a relation the reciprocal of that which subsists between their corresponding integral orders; thus, the relation of a hundred to ten is 10, whilst the relation of a hundredth to a tenth is $\frac{1}{10}$; whence there result regularly descending orders of units which readily unite with the integral orders to form a system of units increasing upwards and decreasing downwards in a tenfold ratio.

This is clearly set forth in the following

201. TABLE OF RELATIONS.

Ten Thousandths	make a Hundredth.
Ten Hundreths	make a Tenth.
Ten Tenths	make a Unit.
Ten Units	make a Ten.
Ten Tens	make a Hundred.
Ten Hundreds	make a Thousand.

NOTE. — Tenths are called decimals of the 1st order; hundredths, decimals of the 2d order; thousandths, decimals of the 3d order, etc.

202. Decimal denominators may be expressed or indicated; but they are commonly understood and not expressed.

203. When expressed, one denominator may be made to suffice for several orders of decimals; as, 1 decimal unit of the 1st order, 1 of the 2d order, and 1 of the 3d order may be written $\frac{111}{1000}$ and read 1 tenth, 1 hundredth, 1 thousandth.

204. Decimals, however, are usually reduced to, and read in terms of, the lowest order; $\frac{111}{1000}$ would generally be read 111 thousandths.

205. For the sake of economy and convenience, it is desirable to omit the denominators. And this can be done; for by writing the decimal orders to the right of the units' place, so that each figure shall stand in the place to the right that corresponds in number with the order of the decimal, the general scheme of the integral notation may be extended to embrace the decimal orders.

The decimal denominators, like the integral orders, will then be designated by the device of place; and the place values, answering to the relation of the orders of units, will increase from right to left, and decrease from left to right, in a tenfold ratio.

206. The **Decimal Point** is a period placed to the left of the tenths' place to determine the decimal orders.

207. A Decimal is a fraction expressed in the scale of 10.

1 decimal unit of the 1st order is written .1, or $\frac{1}{10}$, and read 1 tenth.
3 decimal units of the 1st order is written .3, or $\frac{3}{10}$, and read 3 tenths.
6 decimal units of the 1st order is written .6, or $\frac{6}{10}$, and read 6 tenths.
1 decimal unit of the 2d order is written .01, or $\frac{1}{100}$, and read 1 hundredth.
5 decimal units of the 2d order is written .05, or $\frac{5}{100}$, and read 5 hundredths.

$\left.\begin{array}{l}\text{2 decimal units of the 1st order}\\ \text{5 decimal units of the 2d order}\end{array}\right\}$ is written .25, or $\frac{25}{100}$, and read 25 hundredths.

$\left.\begin{array}{l}\text{2 decimal units of the 1st order}\\ \text{5 decimal units of the 2d order}\\ \text{3 decimal units of the 3d order}\end{array}\right\}$ is written .253, or $\frac{253}{1000}$, and read 253 thousandths.

208. A fraction expressed in the scale of 10 is a decimal whether its denominator be expressed or not. It should not be overlooked that decimals with expressed denominators are written in the scale of 10, that is, in terms of its several orders of units.

For

.25 is $\frac{25}{100}$, which is $\frac{2|5}{10|0}$, or $\frac{2}{10}$, $\frac{5}{100}$;
.253 is $\frac{253}{1000}$, which is $\frac{2|5|3}{10|0|0}$, or $\frac{2}{10}$, $\frac{5}{100}$, $\frac{3}{1000}$;
.00234 is $\frac{234}{100000}$, which is $\frac{2|3|4}{1000|0|0}$, or $\frac{2}{1000}$, $\frac{3}{10000}$, $\frac{4}{100000}$.

Read the following in terms of their several orders of units:

1. $\frac{67895}{100000}$.
Ans. 6 tenths, 7 hundredths, 8 thousandths, 9 ten-thousandths, 5 hundred-thousandths.

2. .50. *Ans.* 5 tenths, 0 hundredths.
3. 2.75. *Ans.* 2 (units), 7 tenths, 5 hundredths.
4. $\frac{536}{100}$. *Ans.* 5 (units), 3 tenths, 6 hundredths.
5. .97.
6. 375.085.
7. 42.973.
8. 81.001.
9. .2750.
10. $5.75.
11. $12.523.
12. .5000.
13. .05.
14. .0005.
15. .0025.
16. 3000.003.
17. $\frac{18795}{10000}$.
18. $\frac{42}{1000}$.
19. $\frac{976}{10000}$.
20. $\frac{7245}{10000}$.
21. $\frac{52128}{10000}$.
22. 5.2128.

209. Omit the denominator and supply the decimal point in the following:

1. $\frac{78934}{100000}$. *Ans.* $\frac{78934}{1\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0}}$, or .78934.

2. $\frac{53}{1000}$. *Ans.* $\frac{053}{1\cancel{0}\cancel{0}\cancel{0}}$, or .053.

Rule. — *Point off as many decimal places from the right of the numerator as there are zeros in the denominator; if there are not so many, supply the deficiency by prefixing zeros.*

3. $\frac{3}{1000}$.
4. $\frac{5}{10000}$.
5. $\frac{185}{1000}$.
6. $\frac{84567}{10000}$.
7. $\frac{64598}{1000}$.
8. $\frac{188}{1000000}$.
9. $\frac{1256}{10000}$.
10. $\frac{76425}{10000}$.
11. $\frac{210}{10000}$.
12. $\frac{76482}{1000}$.
13. $\frac{1}{1000000}$.
14. $\frac{18980}{10000}$.
15. $\frac{3279647}{10000}$.

210. Supply the denominators of the following:

1. .025. *Ans.* $\frac{025}{1000}$, or $\frac{25}{1000}$.

RULE. — *Omit the decimal point and the prefixed zeros (if any), and write for the denominator* 1 *with as many zeros as there are decimal figures (including the prefixed zeros) in the numerator.*

2. .78948.
3. .16179.
4. .0034.
5. .0072.
6. .0081.
7. .00003.
8. .00025.
9. 125.673.
10. 25.340.
11. 11.931.
12. 25.005.
13. 16.034.
14. 29.333.
15. .067315.
16. .00231.
17. .00018.
18. .23456.
19. .00235.

211. Give the denominators of the following at a glance:

1. .0035678950. *Ans.* Ten billion.

EXPLANATION. — Pointing off the decimal into periods of 3 figures each, as in integers, and numerating by periods (units, thousands, millions, billions, units of billions), we find the numerator to be units of billions. The denominator must be one order of unit higher, or ten-billions.

2. .03.
3. .5.
4. .002.
5. .0003.
6. .0000321.
7. .0000001.
8. .23456789.
9. .3157986425.
10. .7600074256.
11. .18924572568.
12. 27.892434561000.
13. 319.176453724568.

212. NUMERATION TABLE.

INTEGRAL ORDERS.											DECIMAL ORDERS.								
Billions.	Hundred-millions.	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.		Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.	Hundred-millionths.	Billionths.
9	4	5	4	3	5	6	4	2	3	.	5	4	2	6	7	8	3	4	5

This number is read, 9 billion, 454 million, 356 thousand, 423, and 542 million, 678 thousand, 345 billionths.

213. TO READ DECIMALS.

1. Read 8375.0025640.

Ans. 8 thousand, 375, and 25 thousand, 640 ten-millionths.

RULE. — *Read the decimal as an integer, and add the name of the right-hand order.*

Read the following:

2. .5.
3. .05.
4. .005.
5. .25.
6. .250.
7. .025.
8. .185.
9. .964.
10. .873.
11. .0005.
12. .0450.
13. .0732.
14. .9657.
15. .8954.
16. .8000.
17. .5700.
18. $ 3.5.
19. $ 3.70.
20. $ 375.643.
21. 223.60043.
22. 125.000750.
23. 3000.003.
24. 500.05.
25. 5678.00000065.
26. 3456.00256832.
27. 2181.00000004.
28. .00000000456.
29. 3796.273391043.
30. 2133.007351924689.
31. 1317.000065430004.
32. 2751.072642100459.
33. 32500.056734200000.

214. To write Decimals.

1. Write 25 ten-millionths decimally. *Ans.* .0000025.

Explanation. — Since the denominator is ten-millions, the numerator must express units of millions; therefore, setting down the decimal point, write 0 millions, 000 thousands, 025; or, .0,000,025.

Note. — There is no higher reason for writing a decimal backward than for writing an integer or spelling a word backward.

Rule. — *Set down the decimal point and write the numerator as an integer, making it express an order of unit one denomination lower than that to be expressed by the denominator (considered as an integer).*

Write decimally:

1. Five and five tenths.
2. Twenty-five and twenty-five hundredths.
3. Seventy-five millionths.
4. Five thousand and five thousandths.
5. 200 million and 200 millionths.
6. 45 million, 32 thousand, 25 hundred-millionths.
7. 375 billion, 264 million, 126 thousand, 237 billionths.
8. Fifty thousand, thirty-four ten-millionths.
9. Seven hundred thousandths.
10. Six hundred fifty-eight thousandths.
11. Eighty-five ten thousandths.
12. Forty-three ten-millionths.
13. Fifty-nine hundred-billionths.
14. 17 thousand, 45 ten-millionths.
15. 10 million and 63 billionths.
16. 40 thousandths.
17. 60 thousand, 72 hundred-thousandths.

18. 43 billionths.

19. 31 million, 375 thousand, 62 hundred-billionths.

20. 31 million, 375 thousand, 62 hundred-thousandths.

21. 50 billion, 65 million, 33 thousand, 47 billionths.

NOTE. — A zero is annexed to a decimal when placed to the right of it; prefixed when placed to the left of it and after the decimal point.

215. Annexing a zero to a decimal does not alter its value, because it multiplies both numerator and denominator by 10; as

$$.5, \text{ or } \tfrac{5}{10} = .50, \text{ or } \tfrac{50}{100} = .500, \text{ or } \tfrac{500}{1000}.$$

216. Prefixing a zero to a decimal divides its value by 10, because it multiplies the denominator by 10; as

$$.5 = \tfrac{5}{10};\ .05 = \tfrac{5}{100};\ .005 = \tfrac{5}{1000}.$$

217. Each removal of the decimal point one place to the left divides the number by 10, because it multiplies the denominator by 10; as

$$12.5 = \tfrac{125}{10};\ 1.25 = \tfrac{125}{100};\ .125 = \tfrac{125}{1000}.$$

218. Each removal of the decimal point one place to the right multiplies the number by 10, because it divides the denominator by 10; as

$$.25 = \tfrac{25}{100};\ 2.5 = \tfrac{25}{10};\ 25. = \tfrac{25}{1}.$$

219. By the removal of the decimal point, find the value of:

1. 2.5 × 10; by 20.
2. 1.72 × 100; by 300.
3. 6.453 × 1000; by 5000.
4. 897.64 ÷ 10; by 40.
5. 673.05 ÷ 100; by 500.
6. 4567.8 ÷ 1000; by 2000.
7. 35.210 × 100; by 700.
8. 459.67 ÷ 10000.

9. .37500 ÷ 100000.
10. 53100* ÷ 100.
11. .825 × 10000.
12. .5 ÷ 10.
13. .5 ÷ 1000.
14. $ 824.673 × 1000.
15. 171.829 × 1000; by 2000.

220. Reduction.

To reduce decimals to higher or lower orders.

1. Reduce .125 to millionths.

PROCESS.
.125 = .125000.

Explanation. — A thousandth of a thousandth is a millionth; therefore multiply both terms by 1000; or, what is the same thing, annex 3 zeros to the numerator.

2. Reduce .75000 to hundredths.

PROCESS.
.75000 = .75.

Explanation. — A hundredth is a 1000 times a hundred-thousandth; therefore divide both terms of the fraction by 1000; or, what is the same thing, reject 3 zeros from the right of the decimal.

Reduce the following:

3. .15 to thousandths; to ten-thousandths.
4. 8 to hundredths. *Ans.* 8.00.
5. .328 to hundred-thousandths; to ten-millionths.
6. 25.16 to ten-thousandths; to thousandths.
7. .3327 to millionths; to ten-millionths.
8. 2.713 to billionths; to millionths.
9. 8.95 to hundred-thousandths; to thousandths.
10. 16.750000 to hundredths; to ten-thousandths.
11. 153.7000 to tenths; to hundredths.
12. 13 to thousandths; to millionths.

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* The decimal point is understood and not expressed after an integer.

221. To reduce decimals to common fractions.

1. Reduce .75 to a common fraction.

PROCESS.

.75, or $\frac{75}{100} = \frac{3}{4}$.

EXPLANATION. — .75 is $\frac{75}{100}$, which, reduced to its lowest terms, is $\frac{3}{4}$.

RULE. — *Omit the decimal point, supply the denominator, and reduce the fraction to its lowest terms.*

Reduce the following to common fractions in their lowest terms:

2. .25.
3. .125.
4. .12.
5. .32.
6. .032.
7. .80.
8. .48.
9. .56.
10. .16.
11. .225.
12. .075.
13. .375.
14. .895.
15. .0075.
16. .00125.

17. $.12\frac{1}{2}$. PROCESS. — $.12\frac{1}{2} = \frac{12\frac{1}{2}}{100} = \frac{\frac{25}{2}}{100} = \frac{25}{200} = \frac{1}{8}$ *Ans.*

18. $.02\frac{1}{2}$.
19. $.33\frac{1}{3}$.
20. $.66\frac{2}{3}$.
21. $.87\frac{1}{2}$.
22. $.62\frac{1}{2}$.
23. $.08\frac{1}{3}$.
24. $.31\frac{1}{4}$.
25. $.16\frac{2}{3}$.
26. $.37\frac{1}{2}$.
27. $.56\frac{1}{4}$.

222. To reduce common fractions to decimals.

1. Reduce $\frac{3}{4}$ to a decimal.

PROCESS.

$\frac{3}{4} = \frac{3.00}{4} = .75.$

EXPLANATION. — $\frac{3}{4}$ is $\frac{1}{4}$ of 3; 3 = 3.0 = 3.00, etc.; $\frac{1}{4}$ of 3.00 = .75.

RULE. — *Annex zeros to the numerator and divide by the denominator. Point off as many decimal places in the quotient as there are zeros annexed.*

Reduce to decimals:

2. $\frac{1}{2}$.
3. $\frac{1}{4}$.
4. $\frac{3}{5}$.
5. $\frac{7}{8}$.
6. $\frac{4}{5}$.
7. $\frac{3}{8}$.
8. $\frac{2}{3}$.
9. $\frac{5}{9}$.
10. $\frac{3}{16}$.
11. $\frac{1}{40}$.
12. $\frac{11}{25}$.
13. $.12\frac{1}{2}$.
14. $.37\frac{1}{2}$.
15. $.27\frac{5}{8}$.
16. $.87\frac{1}{2}$.
17. $.31\frac{1}{4}$.

223. ADDITION AND SUBTRACTION.

1. Add .75, .869, and 3.97.

PROCESS.

$$\begin{array}{r} .75\\ .869\\ 3.97\\ \hline 5.589 \end{array}$$

EXPLANATION. — Write the numbers so that units of the same order shall stand in the same column. Begin at the right and add as in integers; thus 9 thousandths and 0 thousandths = 9 thousandths; write the 9 thousandths under the thousandths' column. 7, 13, 18; 18 hundredths = 1 tenth and 8 hundredths; write the 8 hundredths under the hundredths' column, and carry the 1 tenth to the tenths' column. 1, 10, 18, 25; 25 tenths = 2 (units) and 5 tenths; write the 5 tenths under the tenths' column, bring down the decimal point, and carry the 2 to the units' column. 2, 5; write the 5 units under the units' column. Therefore the required sum is 5.589.

RULE. — *Write the numbers so that the units of the same order shall stand in the same column. Begin at the right, and add as in integers, placing the decimal point at the left of the tenths' order in the sum.*

2. From 65.47 take 23.238.

PROCESS.

$$\begin{array}{r} 65.47\\ 23.238\\ \hline 42.232 \end{array}$$

EXPLANATION. — 8 thousandths cannot be taken from 0 thousandths; therefore borrow 1 hundredth from the 7 hundredths; 1 hundredth = 10 thousandths; 8 thousandths from 10 thousandths leaves 2 thousandths; write the 2 thousandths in the thousandths' place. 3 from 6 leaves 3, and so on as in integers, placing the decimal point at the left of the tenths' order in the difference.

Perform the following indicated operations, and verify:

3. 314.5 + 25.165 + 3.1895 + 256.18.

4. 167.89 + 34.7894 + 83.9456 + 217.35.

5. 189.378 + 825.673 + 27.8956 + 37.0005.

6. 729.360 + 937.612 + 18.2934 + 29.0025.

7. 3764.205 + 3672.0025 + 33.999 + 55.555.

8. 47.0000015 + 10.00055 + 19.00000075.

9. $7.87½ + $93.62½ + $23.37½.

10. .535 − .107 ; .5 − .003 ; .05 − .005.

11. $225.75 − $136.37½ ; 391.125 − 78.094

12. 96.478 − 6.395 ; 279.36 − 37.45.

13. .5786 − .0037 ; 42.105 − 6.789.

14. 189.162 + 876.431 + 16.78 + .95.

15. 29.345 + 17.814 − 35.627 + 82.007.

224. MULTIPLICATION AND DIVISION.

1. Multiply 16.25 by .5.

PROCESS.

$$\begin{array}{r}\left.\begin{array}{r}16.25\\ .5\end{array}\right\} = \dfrac{1625}{100}\times\dfrac{5}{10}=\dfrac{8125}{1000}.\\ \hline 8.125\qquad\qquad\qquad\qquad\qquad\qquad\end{array}$$

EXPLANATION. — Multiply the numerators together for a new numerator, which gives 8125, and the denominators together for a new denominator, which gives 1000; hence the product is $\frac{8125}{1000}$, or 8.125.

Notice that the product of the denominators is found by annexing, or adding the number of zeros in the denominator of the multiplier to the zeros in the denominator of the multiplicand; but the number of zeros in the denominator is the number of decimal places in the fraction decimally expressed. Hence the numerator of the product is obtained by multiplying the numbers together as though they were integers, and the denominator may be obtained by pointing off as many decimal places in the product as there are decimal places in both multiplicand and multiplier.

RULE. — I. *To find the numerator of the product.*

Multiply the numerators together as though they were integers.

II. *To find the denominator of the product.*

Point off as many decimal places in the product as there are decimal places in both factors, prefixing when necessary zeros to supply the deficiency.

2. Divide 1.25 by .25.

PROCESS.

.25)1.25(5
$\underline{\ \ 1.25}$
$= \frac{125}{100} \div \frac{25}{100} = \frac{5}{1}$, or 5.

EXPLANATION. — Divide the numerator of the dividend by the numerator of the divisor for a new numerator, which gives 5; and the denominator of the dividend by the denominator of the divisor, which gives 1; hence the quotient is $\frac{5}{1}$ or 5.

Notice that the quotient of the denominators is found by rejecting or subtracting the number of zeros in the denominator of the divisor from the zeros in the denominator of the dividend; but the number of zeros in the denominator is the number of decimal places in the fraction decimally expressed. Hence the numerator of the quotient is obtained by dividing the numerator of the dividend by the numerator of the divisor, and the denominator may be obtained by pointing off as many decimal places in the quotient as the decimal places in the dividend exceed those in the divisor.

NOTE. — If the decimal places, or zeros, in the denominator of the dividend be fewer than those in the divisor, make them at least equal by annexing zeros, or reducing the dividend to lower terms.

3. Divide .125 by 2.5.

PROCESS.

2.5).125(.05
$\underline{\ \ .125}$

EXPLANATION. — There are 3 decimal places in the dividend, and 1 in the divisor; consequently there will be $3 - 1 = 2$ decimal places in the quotient. Divided as integers the quotient is 5, or only 1 figure; prefix a zero to supply the deficiency.

4. Divide 12.5 by .25.

PROCESS.

.25)12.50(50
$\underline{\ \ 125\ }$
0

EXPLANATION. — Annex a zero to the dividend and divide. There will be $2 - 2 = 0$ decimal places in the quotient; that is, the quotient will be an integer.

RULE. — I. *To find the numerator of the quotient.*

Divide the numerator of the dividend by the numerator of the divisor.

II. *To find the denominator of the quotient.*

Point off as many decimal places in the quotient as the decimal places in the dividend exceed those in the divisor; when necessary, prefix zeros to supply the deficiency.

Perform the indicated operations and verify:

5. 625 × .25.
6. 336 × .125.
7. 25.65 ÷ 5.7.
8. 35.91 ÷ .285.
9. 834 ÷ .47.
10. .1344 ÷ 29.
11. 23.36 ÷ 16.
12. 5.0825 ÷ 1.07.
13. 196.5 × 3.7.
14. 18.75 × .63.
15. 734.5 × 25.
16. 93.72 × 18.75.
17. 44.16 × .001.
18. 33.54 ÷ .01.
19. 35.643 ÷ .001.
20. 1361.25 ÷ 3.3.
21. 16.65 ÷ 3.7.
22. 1432.25 ÷ 425.
23. 5.169 ÷ 1723.
24. 817.75 ÷ 12½.
25. 6542 × 12½.

225. Examples for Review.

1. A farmer sells 7 pigs at $1.25 each. What does he receive for them?
2. A man owing his tailor $87.37, hands him a $100 bill. How much change is due him?
3. What would be the cost of 125 pounds of tea, at 87 cents a pound?
4. How many pounds of tea, at 75 cents a pound, can be bought for $11.25?
5. What will 14.5 yards of linen cost at $.655 per yard?
6. A man had $100 on the first of the month, and $13.87 at the end of the month. What amount did he spend that month?
7. How much must be added to $75.758 to make $100?
8. A owes the following sums: $25.13, $17.853, $23.105, $6.95, $37.785. What amount must he draw out of the bank in order to settle his bills?

9. If $.06 be divided among 12 boys, what will each boy receive?

10. Two men start from the same point; one travels 52.39 miles north, and the other 39.52 miles south. How far apart are they?

11. What decimal fraction is equal to $\frac{7}{8}$?

12. What common fraction is equivalent to .0375?

13. The product of two numbers is 2.26, and one of the numbers is .25. What is the other?

14. A man's income is $2500 a year; he spends $187.50 a month. How much does he lay by?

15. At $1.25 a foot, what would it cost to have a well dug 87.5 feet deep?

16. At $1.12½ a day, how many days will it take to earn $28.12½?

17. A man sold a piece of property for $3175.25, which was $215.87 more than it cost him. What was the cost of the property?

18. A farmer had a horse worth $135.50, which he exchanged for a pair of mules worth $75.15 each, paying the difference in money. How much money did he pay?

19. What will 25.6 pounds of coffee cost, at $.185 a pound?

20. A merchant sold 5 pieces of cloth; the first contained 90.000005 yards; the second, 127.004 yards; the third, 58.0025 yards; the fourth, 64.289 yards; the fifth, 42.006 yards. How many yards, in all, did he sell?

21. What is the difference between 8 hundredths and 8 thousandths?

22. A drover bought 25 mules, at \$ 62.87½ each, and sold them at \$ 70 each. How much did he gain?

23. What will 14.5 bushels of potatoes cost, at \$ .625 a bushel?

24. At \$ 12.78 each, how many cows can be bought for \$ 242.82?

25. A travelled 522.35 miles in 15.5 days. What distance did he average per day?

26. \$ 54.375 + \$ 243.405 + \$ 324.095 + \$ 47.75 + \$ 83.005 + \$ 99.099 – \$ 62.008 + \$ 53.025 + \$ 107.777 + \$ 96.425 + \$ 85.038 = ?

27. If sugar is worth \$ 7.25 a barrel, how many barrels can be bought for \$ 112.375?

28. How many sacks would be required to hold 102.66 bushels of wheat, each sack holding 3.54 bushels?

29. If 37.6 acres of land yield 1097.544 bushels of wheat, what is the average yield per acre?

30. A mechanic earns \$ 20 a week, and his expenses amount to \$ 17.25 a week. How much will he save in a year, or 52 weeks?

31. A man exchanged three town lots, valued respectively at \$ 800.90, \$ 535.75, \$ 1025.48, for a farm worth \$ 2500, paying the difference in money. How much did he pay?

32. Mr. Sams deposits in a bank at different times the following sums: \$ 74.15, \$ 127.94, \$ 19.475, \$ 215.13, \$ 103.875, and draws out at different times, \$ 75.65, \$ 150.73, \$ 12.25, \$ 68.91. What is his bank balance?

33. A man sold 147.5 acres of land, at \$ 5.75 an acre, and invested the money in petroleum at 27½ cents a gallon. How much petroleum did he buy?

34. What number added to the sum of .25 + .1 + .125 + .40 will equal 1?

35. A merchant bought 8 bales of sheeting, each containing 20 pieces, and each piece 47 yards; the whole cost him $ 1128. What did it cost him per yard?

36. A grain dealer owning 58500 bushels of wheat, sold $\frac{1}{8}$ of it at $ .375 per bushel, and the remainder at $ .823 per bushel. What did he receive for his wheat?

37. A farmer sold 1263 pounds of pork, at 5 cents a pound, and took in payment flour, at $ 4.21 a barrel. How much flour did he get?

38. A man sold .15 of an estate to one person, .19 to another, .3 to another, and .255 to another. What part of the estate remained unsold?

39. A man travelled 420.75 miles in 6 days; on each of the first 5 days he went an equal distance; on the sixth day he went 93.25 miles. How far did he go each day?

CHAPTER IX.

RATIO AND PROPORTION.

226. **Ratio** is the measure of the relation which one quantity or number bears to another of the same kind. Whence every number, integral or fractional, expresses a ratio. An integer expresses the ratio of the number to 1, as 5 is 5 times 1, or 5 ones, which may be written $\frac{5}{1}$; and a fraction expresses the ratio of the numerator to the denominator; $\frac{3}{4}$ expresses the relation of 3 to 4.

227. The ratio of one number to another is found by dividing the number compared by the number with which it is compared.

228. Ratio is sometimes expressed by writing the numerator to the left and the denominator to the right of a colon; $\frac{3}{4}$ may be written $3:4$. When thus separated by a colon, the numerator is called the antecedent and the denominator the consequent.

229. An **Equation** is the expression of equality between two quantities; as $5 \times 3 = 15$.

The quantity to the left of the sign of equality is called the **First Member**, and that to the right the **Second Member** of the equation.

230. A **Proportion** is a special form of the equation. It expresses the equality of two ratios; as $\frac{3}{4} = \frac{6}{8}$. Every equation may be expressed in the form of a proportion, and every proportion in the general form of the equation.

231. If two ratios are equal, their reciprocals are also equal; $\frac{3}{4} = \frac{6}{8}$, $\frac{4}{3} = \frac{8}{6}$. Whence the inversion of a proportion does not destroy its proportionality.

232. The proportion $\frac{3}{4} = \frac{6}{8}$ is also written $3 : 4 = 6 : 8$, or $3 : 4 :: 6 : 8$. It may be read 3 is to 4 as 6 is to 8; or as 3 is to 4, so 6 is to 8; or 3 is that part of 4 that 6 is of 8; or, by inversion, 4 is that part of 3 that 8 is of 6.

The 1st and 4th terms of a proportion are called the **Extremes**; the 2d and 3d, the **Means.**

233. There are four readings, or interpretations, of the proportion $3 : 4 :: 6 : 8$, which give rise to as many equivalent expressions:

1st, 3 is that part of 4 that 6 is of 8, viz., $\frac{6}{8}$; or $3 = \frac{6}{8}$ of $4 = \frac{6 \times 4}{8}$.

2d, 4 is that part of 3 that 8 is of 6, viz., $\frac{8}{6}$; or $4 = \frac{8}{6}$ of $3 = \frac{8 \times 3}{6}$.

3d, 6 is that part of 8 that 3 is of 4, viz., $\frac{3}{4}$; or $6 = \frac{3}{4}$ of $8 = \frac{3 \times 8}{4}$.

4th, 8 is that part of 6 that 4 is of 3, viz., $\frac{4}{3}$; or $8 = \frac{4}{3}$ of $6 = \frac{4 \times 6}{3}$.

234. In 1st example, 3 is the quotient of 6×4 divided by 8; but the quotient multiplied by the divisor equals the product. Hence, $3 \times 8 = 6 \times 4$. 8 and 3 are extremes; 6 and 4 are means. Each of the examples yields the same equation.

Whence the following principles:

The product of the extremes equals the product of the means.

Either extreme is equal to the product of the means divided by the other extreme.

Either mean is equal to the product of the extremes divided by the other mean.

235. Or more generally: Let the proportion 3 : 4 :: 6 : 8 be expressed in its original form $\frac{3}{4}=\frac{6}{8}$, and be reduced to equivalent fractions having a common denominator; as $\frac{3\times 8}{48}=\frac{4\times 6}{48}$. Since these fractional ratios are equal and have a common denominator, their numerators (3×8 and 4×6) are equal; but the numerator of the 1st ratio is the product of the extremes, and the numerator of the 2d ratio is the product of the means.

Whence again:

The product of the extremes equals the product of the means.

Either extreme is equal to the product of the means divided by the other extreme.

Either mean is equal to the product of the extremes divided by the other mean.

236. The proportion 3 : 4 :: 6 : 8 may therefore be expressed in the following forms of the equation:

$$3\times 8=4\times 6;\ \frac{4\times 6}{8}=3;\ \frac{4\times 6}{3}=8;\ \frac{3\times 8}{4}=6;\ \frac{3\times 8}{6}=4.$$

237. Examples for Practice.

1. What is the ratio of 1 to 5? *Ans.* $\frac{1}{5}$.
2. What is the ratio of 5 to 1? *Ans.* $\frac{5}{1}$.
3. What is the ratio of 3 to 5?

Solution.—Since 1 is $\frac{1}{5}$ of 5, 3 is 3 times $\frac{1}{5}$ of 5, or $\frac{3}{5}$.

4. What part of 3 is 2?
5. What part of 8 is 7?
6. What part of 12 is 9?
7. What part of $60 is $20?

8. What part of 72 pounds is 60 pounds?

9. What part of 81 horses is 63 horses?

10. What part of 5 is $\frac{3}{4}$?

SOLUTION. — 1 is $\frac{1}{5}$ of 5, and $\frac{3}{4}$ of 1 is $\frac{3}{4}$ of $\frac{1}{5}$ of 5 $= \frac{3}{20}$.

11. What is the ratio of $\frac{2}{3}$ to 9?

12. What is the ratio of $\frac{5}{9}$ to 15?

13. What is the ratio of $\frac{3}{7}$ to 18?

14. What part of $\frac{3}{4}$ is $\frac{2}{3}$?

SOLUTION. — $\frac{3}{4} = \frac{9}{12}$; $\frac{2}{3} = \frac{8}{12}$; $\frac{8}{12}$ is of $\frac{9}{12}$ as 8 is of 9, or $\frac{8}{9}$, or $\frac{\frac{2}{3}}{\frac{3}{4}} = \frac{8}{9}$.

15. What is the ratio of $\frac{3}{5}$ to $\frac{7}{9}$?

16. What part of $\frac{11}{12}$ is $\frac{5}{6}$?

17. What is the ratio of $\frac{3}{8}$ to $\frac{7}{15}$?

238. Interpret and write each of the following as a fractional proportion:

1. $4 : 12 :: 15 : 45$. *Ans.* $\frac{4}{12}$ $(\frac{1}{3})$ of $45 = 15$; $\frac{12}{4}$ of $15 = 45$; etc.

2. $5 : 15 : : 8 : 24$.

3. $3 : \ 5 : : 18 : 30$.

4. $7 : 21 : : 21 : 63$.

5. $9 : 36 : : 15 : 60$.

6. $12 : 18 : : 32 : 48$.

7. $27 : 3 : : 9 : 1$.

8. $63 : 72 : : 14 : 16$.

239. Write each of the above as an equation; as

$$4 : 12 : : 15 : 45 = 4 \times 45 = 12 \times 15.$$

240. Supply the omitted term, which is represented by x, in each of the following proportions:

1. $16 : 12 :: 72 : x$.

Solution. — x is that part of 72 that 12 is of 16, viz., $\frac{12}{16}$. Whence $x = \frac{12}{16}$ of $72 = 54$; or, since the product of the extremes equals the product of the means, $16x = 12 \times 72$; $x = \frac{12 \times 72}{16} = 54$.

2. $18 : 27 :: 60 : x$.
3. $32 : 64 :: x : 90$.
4. $25 : 35 :: 165 : x$.
5. $10 : 16 :: 30 : x$.
6. $9 : 25 :: 72 : x$.
7. $45 : 9 :: x : 25$.
8. $48 : x :: 7 : 8$.
9. $54 : x :: 48 : 24$.
10. $17 : 51 :: x : 72$.
11. $69 : 23 :: 81 : x$.
12. $\frac{2}{3} : \frac{3}{4} :: 24 : x$.
13. $\frac{3}{5} : \frac{7}{8} :: 10 : x$.
14. $12\frac{1}{2} : 37\frac{1}{2} :: x : 36$.
15. $x : 33\frac{1}{3} :: 16 : 32$.

241. Every solution of a problem requiring the comparison of numbers involves a proportion, whether the proportion is formally stated or not. Notice the implicit proportion in the solution of these simple examples:

1. If 1 yard of cloth cost 6 cents, what will 5 yards cost?

Solution. — If 1 yard cost 6 cents, 5 yards (which is 5 times 1 yard) will cost 5 times 6 cents = 30 cents; which is to say, as 5 yards is to 1 yard, so the price of 5 yards is to 6 cents (the price of 1 yard); or, formally stated,

5 yards : 1 yard : : x cents : 6 cents.

2. If 5 yards of cloth cost 30 cents, what will 1 yard cost?

Solution. — If 5 yards cost 30 cents, 1 yard (which is $\frac{1}{5}$ of 5 yards) will cost $\frac{1}{5}$ of 30 cents = 6 cents; which is to say, as 1 yard is to 5 yards, so the price of 1 yard (x cents) is to the price of 5 yards (30 cents); or, formally stated,

1 yard : 5 yards : : x cents : 30 cents.

242. PRIMARY PROPORTION, OR ANALYSIS.

1. If 3 yards of cloth cost 18 cents, what will 5 yards cost?

EXPLANATION. — This problem may be resolved into two problems, not distinct and separable, but interdependent; as, (*a*) If 3 yards of cloth cost 18 cents, what will 1 yard cost? (*b*) If 1 yard cost x (6 cents), what will 5 yards cost?

SOLUTION. — (*a*) If 3 yards cost 18 cents, 1 yard will cost $\frac{1}{3}$ of 18 cents = 6 cents. } 1st step.

(*b*) If 1 yard cost 6 cents, 5 yards will cost 5 times 6 cents = 30 cents. } 2d step.

Or, formally stated,

(*a*) 3 yards : 1 yard : : 18 cents : *Ans.* (= 6).

(*b*) 1 yard : 5 yards : : 6 cents : 2d *Ans.* (= 30 cents).

In this solution the numbers 5 and 3 are compared through their relation to the unit 1, the relation being made *explicit.* For this reason the problem resolves, analyzes, itself into two problems, and the method of solution is therefore commonly known as analysis.*

243. SIMPLE PROPORTION.

1. If 3 yards of cloth cost 18 cents, what will 5 yards cost?

SOLUTION. — Since 5 yards are $\frac{5}{3}$ of 3 yards, they will cost $\frac{5}{3}$ as much, or $\frac{5}{3}$ of 18 cents = 30 cents. Or, formally stated,

3 yards : 5 yards : : 18 cents : x cents.

$$x = \tfrac{5}{3} \text{ of } 18 = 30.$$

Or, multiplying the extremes together,

$$3x = 90.$$
$$x = 30.$$

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* This is obviously the origin of the use of this term and completely answers the objection urged by some that since the first step of the process is analytical and the second synthetic, the method should be known as analytico-synthetical.

In this solution the numbers 5 and 3 are compared directly with each other, their relation to the unit 1 being only implicitly recognized (as in the comparison of a multiple to its factor, 8 to 4 or 6 to 3). This method is known as simple proportion or sometimes (when expressed in the fractional form) as fractional proportion.

244. FORMAL PROPORTION.

1. If 3 yards of cloth cost 18 cents, what will 5 yards cost?

From the above it will be seen that if the steps in the analytical process be formalized, they will be expressed thus:

3 yards : 1 yard : : 18 cents : *Ans.* (= 6 cents);
1 yard : 5 yards : : 6 cents : 2d *Ans.* (= 30 cents);

while the process of the simple proportion formalized gives

3 yards : 5 yards : : 18 cents : x cents.

SOLUTION. — Interpreting the last expression, we have, x cents is that part of 18 cents that 5 yards is of 3 yards, viz., $\frac{5}{3}$; that is, x cents $= \frac{5}{3}$ of 18 cents $= 30$ cents, which is its original fractional form.

TO THE TEACHER. — When the pupil has reached Compound Proportion, it will be well to have him return to this article, that he may see that the formal expression of the simple proportion may be derived from the above set of primary proportions by multiplying the corresponding terms together, just as the compound proportion is reduced to the form of a simple proportion; in other words, that there is no principle in compound proportion that is not implicitly involved in simple proportion.

It is to be hoped that the teacher will see to it that these several methods of solution are presented in the order of their evolution and therefore in the order in which the child-mind naturally unfolds. Too great attention cannot well be bestowed upon this subject; for proportion in one form or another is the instrument of all mathematical investigation. Proportion bears the same relation to arithmetic that its more general form, the equation, sustains to algebra. It is, therefore, a gross misapprehension of the function of proportion to co-ordinate it with the several subordinate subjects of arithmetic; as interest, discounts, commission, etc. The examples that follow are many and varied. Let the class solve them first by the primary, or analytical, method, then in review by the fractional method, and finally by the formal method. Analysis is the child method; proportion proper the adult method. It is the highest end of the school to develop manhood, which can be accomplished only by a graduated advance in the methods used to elicit the child's activities.

245. Some of the following problems require several steps in their solution, not all of which involve the comparison of numbers. That is, one or more of the steps are accomplished by addition or subtraction. On this account, this class of problems is usually excluded from proportion. This, it seems to us, is a mistake; for, if only one of many steps in the solution of a problem involves comparison, the example properly belongs under proportion. Let an example illustrate:

If A can do a piece of work in 5 days, and B can do the same work in 6 days, how long will it take A and B, working together, to do the work?

Solution. — 1st step. To find what part of the work A can do in 1 day:

If A can do the work in 5 days, in 1 day (which is $\frac{1}{5}$ of 5 days) he can do $\frac{1}{5}$ of the work (proportion).

2d step. To find what part of the work B can do in 1 day:

If B can do the work in 6 days, in 1 day (which is $\frac{1}{6}$ of 6 days) he can do $\frac{1}{6}$ of the work (proportion).

3d step. To find what part of the work both A and B can do in 1 day:

If A does $\frac{1}{5}$ of the work in 1 day, and B does $\frac{1}{6}$ of the work in 1 day, both, working together, can do $\frac{1}{5} + \frac{1}{6}$ or $\frac{11}{30}$ of the work in 1 day (addition).

4th step. To find how many days it will take both to do the whole work:

If it takes both 1 day to do $\frac{11}{30}$ of the work, to do $\frac{30}{30}$ of the work (which is $\frac{30}{11}$ of $\frac{11}{30}$) it will take $\frac{30}{11}$ days $= 2\frac{8}{11}$ days, or $\frac{11}{30}$ work : $\frac{30}{30}$ work :: 1 da. : x da. ($= 2\frac{8}{11}$ da.) (proportion).

246. Examples.

1. If 4 yards of cloth cost 60 cents, what will 30 yards of the same cloth cost?
2. If 27 yards of cloth cost 81 dollars, how many yards of the same cloth can be bought for $ 45?
3. The rent of 50 acres of land is $ 500. How many acres, at the same rate, can be rented for $ 100?
4. How long will a person be in saving $ 1000, if he lay by 50 cents a week?
5. If an income of $ 5000 is taxed $ 100, what should an income of $ 8000 be taxed?
6. Find the value of 3 ton 5 cwt. 3 qr. of hay at $ 1.25 a cwt.
7. If 20 sheep cost $ 60, what will 40 sheep cost?
8. If a man travels 210 miles in 7 days, how far will he travel in 5 days?
9. If 6 barrels of flour cost $ 24, what will 9 barrels cost?
10. If 5 cows cost $ 75, how much will 12 cows cost?
11. If the wool from 15 sheep brings $ 75, what will the wool from 45 sheep bring?
12. If a man earn $ 1600 a year, what does he earn in 9 months?
13. A man travels 25 miles in 5 hours. How far does he travel in 8 hours?
14. If 16 barrels of apples cost $ 72, what will 30 barrels cost?
15. If 8 pounds of butter cost $ 1.20, how much will 15 pounds cost?

16. At 5 cents a pound, what will a bale of cotton weighing 500 pounds bring?

17. If 9 barrels of flour cost $ 32, what will 28 barrels cost?

18. If 12 horses eat 20 bushels of oats in a week, how many bushels will 45 horses eat in the same time?

19. If 63 gallons of wine cost $ 98.50, what will 45 gallons cost at the same rate?

20. If a barrel of flour support 12 men 25 days, how long will it support 8 men?

21. If 3 yards of cloth cost $ 12, what will 7 yards cost?

22. If 5 pounds of rice cost 15 cents, what will 16 pounds cost?

23. If a staff 9 ft. high cast a shadow 4 ft. long, what will be the length of the shadow cast by a post 27 feet high?

24. If 5 yards of calico cost 25 cents, what will 17 yards cost?

25. If 3 men can dig a trench in 12 days, how long would 9 men take to dig it?

26. If 8 yards of silk cost $ 24, how many yards can be bought for $ 42?

27. Ten days' work, at 60 cents a day, will pay for how many pair of shoes, at $ 2 a pair?

28. If $\frac{4}{5}$ of a man's income is $ 5985.80, what is his entire income?

29. A piece of land of a certain length, and 25 rods in width, contains $2\frac{1}{2}$ acres. How much would there be in a piece of the same length $53\frac{1}{8}$ rods wide?

30. If 40 yards of carpeting, $\frac{7}{8}$ of a yard wide, be required to cover a floor, how many yards would be required if the carpeting were 1 yard wide?

31. If 5½ shares in a street railroad are worth $ 412.50, what will 6⅔ shares be worth?

32. If ⅔ of a bushel of wheat cost 46 cents, what is the cost of ½ bushel?

33. If 2½ acres of land cost $ 63, what will 22½ acres cost?

34. If 25 pounds of tobacco cost $ 7.50, how much will 185 pounds cost?

35. If 9 men reap a field of oats in 20 days, how many men would be required to reap it in 30 days?

36. If a pole 8 ft. high cast a shadow 12 ft. long, what will be the length of the shadow cast by a tree 80 ft. high?

37. If 3 men can do a piece of work in 8 days, how long will it take 4 men to do it?

38. If 15 men can do a piece of work in 4 days, how long will it take 5 men to do it?

39. If 3 men can do a piece of work in 9 days, how long will it take 15 men to do it?

40. What would provision 5 men for 40 days, would subsist how many men for 8 days?

41. A job that would employ 12 men 6 days, would take 9 men how many days?

42. If 2 pounds of steak cost 25 cents, how many pounds may be bought for 37½ cents?

43. A quantity of flour that will last 16 men 25 days, will last 80 men how many days?

44. If from my age you take ⅓ and ¼ of my age, the remainder will be 10 years. How old am I?

45. What number is that to which if its half be added the sum will be 45?

46. If a ferry-boat cross a river 18 times in 5 hours, in how many hours will it cross 4 times?

47. What will 269 yards of cloth amount to, at the rate of $ 100 for 30 yards?

48. If a post 5 ft. high cast a shadow 3 ft., what is the height of a steeple which at the same time casts a shadow 176 ft?

49. At the rate of $ 182.50 for 8 acres of land, what is the value of 12.7 acres?

50. In how many minutes will a locomotive run 49.9 miles, if it runs 2.5 miles in 5.75 minutes?

51. If $\frac{7}{8}$ of a ship cost $ 15000, what does $\frac{1}{6}$ of it cost?

52. If 9 yards of cloth, $\frac{3}{4}$ of a yard wide, will make a dress, how many yards $\frac{5}{4}$ wide will make the same dress?

53. A company of men can do a piece of work in 155 days by working 12 hours a day. In what time will they do the work if they work only 5 hours a day?

54. If 110 yards of paper, 32 inches wide, will cover the walls of a room, how many yards 24 inches wide will cover the same walls?

55. If the hind wheels of a coach, which are 180 inches in circumference, will turn round 4825 times in running a certain distance, how many times will the front wheels revolve, they being 145 inches in circumference?

56. A rides $6\frac{7}{8}$ miles an hour, and accomplishes a certain journey in $14\frac{5}{12}$ days. In what time will B, who rides only $4\frac{9}{10}$ miles an hour, perform the same journey?

57. A owned $\frac{5}{24}$ of a ship, which he sold for $ 3650; and B owns $\frac{3}{10}$ of it, which he wishes to sell at the same rate. What must be B's price?

58. A garrison of 900 men have provisions for 4 months. How many men must leave the garrison, that the provisions may last the remainder 9 months?

59. After a tract of land had been equally divided among 16 owners, one of them sold $\frac{2}{5}$ of his share at $ 5 an acre, and received $ 444. How much land was in the whole tract?

60. A and B hired a pasture for $ 49.50, in which A pastured 13 cows, and B 19. What must each pay?

61. If 3 horses cost as much as 4 mules, how many mules must be exchanged for 18 horses?

62. A guardian paid one of his wards $ 3500 for $ 2500, which he had in his use 8 years. At the same rate, what should he pay another for the use of $ 3500 for the same length of time?

63. If $13\frac{1}{2}$ yards of cloth cost $ 6.48, what would $8\frac{3}{4}$ yards cost at the same rate?

64. If by travelling $13\frac{5}{8}$ hours a day, a man can finish his journey in $35\frac{1}{2}$ days, how long will it take him if he travel $11\frac{9}{10}$ hours a day?

65. An engine of 40-horse power draws a train of cars at the rate of 32 miles an hour. At what rate would it be drawn by an engine of 35-horse power?

66. A coach wheel, 8 feet 3 inches in circumference, revolves 3496 times in performing a certain journey. What would be the circumference of a wheel which made 3762 revolutions in the same distance?

67. How many dozen handkerchiefs, at 50 cents apiece, should be given for 36 pair of shoes, at $ 2.50 a pair?

68. If $22\frac{1}{2}$ acres of land cost $ 567, how many acres can be bought for $ 63?

46. If a ferry-boat cross a river 18 times in 5 hours, in how many hours will it cross 4 times?

47. What will 269 yards of cloth amount to, at the rate of $ 100 for 30 yards?

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68. If $22\frac{1}{2}$ acres of land cost $ 567, how many acres can be bought for $ 63?

90. If 60 men can do a piece of work in 8 days, how many men can do the same work in 20 days?

91. If 10 cows eat 16 tons of hay, how many cows will 112 tons supply for the same length of time?

92. If a man owning $\frac{3}{5}$ of a farm sells $\frac{2}{3}$ of his share for $680, what is the whole farm worth at the same rate?

93. A and B can do a piece of work in 5 days; A alone can do it in 7 days. In what time can B do the work?

94. After A has travelled 51 miles, B sets out to overtake him, and travels 19 miles while A travels 16. How many miles will each have travelled before B overtakes A?

95. A person having spent in one year all of his income and $\frac{1}{4}$ as much more, found that by saving $\frac{1}{25}$ of his income afterwards, he could, in 4 years, make good the deficiency and have $20 left. What was his income?

96. The hour and minute hands of a watch are together at 12 o'clock. When are they next together?

97. If the time past noon is equal to $\frac{4}{5}$ of the time to midnight, what o'clock is it?

98. A having asked B's age, B replied: "Your age is 12 years, to which if $\frac{5}{8}$ of both our ages be added to your age, the sum will equal my age." What was B's age?

99. A and B together consume a certain quantity of food in 15 days. It will last A alone 27 days. How long will it last B?

100. A can do a piece of work in 10 days, and B in 13 days. In what time will both working together do it?

101. A cistern, containing 60 gallons of water, has three unequal faucets for discharging it. The greatest will discharge it in 1 hour; the second, in 2 hours; the third, in 3 hours. In what time will the cistern be emptied if all three run at the same time?

102. A man bought some lemons at 2 cents each, and $\frac{3}{4}$ as many at 3 cents each. He sold them all at the rate of 5 cents for 2, thus gaining 25 cents. How many lemons did he buy?

103. A farmer sold some cows and calves for $240. The calves he sold at $8, and the cows at $16 apiece. There were 3 times as many calves as cows. What was the number of each?

104. A man, having $100, spent a part of it, and afterward received 5 times as much as he had spent, when he found his money was double what it was at first. How much did he spend?

105. A hare has 50 leaps the start of a greyhound and takes 4 leaps while the hound takes 3; but 2 of the hound's leaps equal 3 of the hare's. How many leaps must the hound take to overtake the hare?

106. A has 100 yards the start of B; but B runs 14 yards while A runs 10. How far must B run to overtake A?

107. A man was hired for a term of 50 days, on condition that for every day he worked he should receive 75 cents, and for every day he was idle he should pay 25 cents for his board. At the expiration of the time he was paid $27.50. How many days was he idle?

108. A man when married was 3 times as old as his wife. 15 years afterward he was but twice as old as his wife. At what age was each married?

CHAPTER X.

COMPOUND PROPORTION.

247. A **Compound Ratio** is the (indicated) product of two or more simple ratios; or, since a ratio is a fraction, a compound ratio is a compound fraction.

248. A **Compound Proportion** is a proportion, one or both of whose numbers are compound ratios.

249. If the corresponding terms of two or more simple proportions be multiplied together, the result will be a proportion; for the products of equal are equals; as,

$$\begin{array}{rl} (1) & 3 : \ 4 :: \ 6 : \ 8 \\ (2) & \underline{2 : \ 4 :: \ 5 : 10} \\ & 6 : 16 :: 30 : 80 \end{array} \qquad \text{Or,} \quad \begin{array}{c} \frac{3}{4} = \frac{6}{8} \\ \underline{\frac{2}{4} = \frac{5}{10}} \\ \frac{3}{4} \times \frac{2}{4} = \frac{6}{8} \times \frac{5}{10} \\ \frac{6}{16} = \frac{30}{80}. \end{array}$$

250. When the multiplication of two or more proportions together is performed, the resulting products form a simple proportion. $\frac{3}{4}$ of $\frac{2}{4} = \frac{6}{8}$ of $\frac{5}{10}$ is a compound proportion; $\frac{6}{16} = \frac{30}{80}$ is a simple proportion. It is therefore incorrect to define a compound proportion as the product of two or more simple proportions.

In the practical solution of problems, the proportion generally occurs in this form:

$\frac{3}{4}$ of $\frac{2}{4} = \frac{30}{80}$; or, $\left.\begin{array}{c} 3 : 4 \\ 2 : 4 \end{array}\right\} :: 30 : 80$, which is brought to a simple proportion by reducing the first number to a simple ratio; $6 : 16 :: 30 : 80$.

251. Problems in compound proportion may be solved by any one of three methods: viz., by primary proportion, or analysis; by simple proportion; or by compound proportion; and the process of each may be formalized or not.

252. 1. If 3 horses eat 45 bundles of oats in 3 days, how many bundles of oats will supply 5 horses 2 days?

For convenience, express the problem in this abbreviated form:

If 3 horses in 3 days eat 45 bundles,
5 horses in 2 days will eat (?) bundles.

An ultimate analysis resolves this problem into four problems in primary proportion. They are:

a. If 3 horses in 3 days eat 45 bundles,
1 horse in 3 days will eat (?) bundles (= 15 bundles).

b. If 1 horse in 3 days eat 15 bundles,
5 horses in 3 days will eat (?) bundles (= 75 bundles).

c. If 5 horses in 3 days eat 75 bundles,
5 horses in 1 day will eat (?) bundles (= 25 bundles).

d. If 5 horses in 1 day eat 25 bundles,
5 horses in 2 days will eat (?) bundles (= 50 bundles).

Solution. — By primary proportion, or analysis.

a. If 3 horses in 3 days eat 45 bundles,
1 horse in 3 days will eat $\frac{1}{3}$ of 45 bundles = 15 bundles.

b. If 1 horse in 3 days eats 15 bundles,
5 horses in 3 days will eat 5 × 15 bundles = 75 bundles.

c. If 5 horses in 3 days eat 75 bundles,
5 horses in 1 day will eat $\frac{1}{3}$ of 75 bundles = 25 bundles.

d. If 5 horses in 1 day eat 25 bundles,
5 horses in 2 days will eat 2 × 25 bundles = 50 bundles.

Or, formally stated:

3 horses : 1 horse :: 45 bundles : *Ans.* (=15 bundles).
1 horse : 5 horses :: 15 bundles : 2d *Ans.* (= 75 bundles).
3 days : 1 day :: 75 bundles : 3d *Ans.* (= 25 bundles).
1 day : 2 days :: 25 bundles : 4th *Ans.* (= 50 bundles).

Notice that there are four analytical steps, or primary proportions, in the above solution, and that the result reached in each step is a proximate answer, and always of the same denomination as the final answer.

253. By comparing the numbers of the same denomination *directly* with each other and not explicitly through their relation to the unit 1, the four problems will be reduced to two and solved as two interdependent simple proportions.

If 3 horses in 3 days eat 45 bundles (oats),
5 horses in 2 days will eat (?) bundles.

A proximate analysis resolves this problem into two problems in simple proportion. They are:

a. If 3 horses in 3 days eat 45 bundles,
5 horses in 3 days will eat (?) bundles (= 75 bundles).

b. If 5 horses in 3 days eat 75 bundles,
5 horses in 2 days will eat (?) bundles (= 50 bundles).

Solution. — *a.* If 3 horses in 3 days eat 45 bundles, 5 horses (which are $\frac{5}{3}$ of 3 horses) in the same time 3 days will eat $\frac{5}{3}$ as many bundles, or $\frac{5}{3}$ of 45 bundles = 75 bundles.

b. If 5 horses in 3 days eat 75 bundles, 5 horses in 2 days (which is $\frac{2}{3}$ as long) will eat $\frac{2}{3}$ as many bundles, or $\frac{2}{3}$ of 75 bundles = 50 bundles.

Or in better form:

a. If 3 horses in 3 days eat 45 bundles,
5 horses in 3 days will eat $\frac{5}{3}$ of 45 bundles = 75 bundles.

b. If 3 horses in 3 days eat 75 bundles,
5 horses in 2 days will eat $\frac{2}{3}$ of 75 bundles = 50 bundles.

As formally stated:

3 horses : 5 horses : : 45 bundles : *Ans.* (= 75 bundles).
3 days : 2 days : : 75 bundles : 2d *Ans.* (= 50 bundles).

254. The problem may be solved as one problem.

1. If 3 horses in 3 days eat 45 bundles oats,
   5 horses in 2 days will eat (?)

SOLUTION. — $\frac{5}{3}$ as many horses in $\frac{2}{3}$ as long a time will eat $\frac{2}{3}$ of $\frac{5}{3}$ of 45 bundles = 50 bundles. This is a real compound proportion, which may readily be reduced to a simple proportion by reducing the compound ratio, or fraction, $\frac{2}{3}$ of $\frac{5}{3}$, to the simple ratio $\frac{10}{9}$. The proportion then becomes $\frac{10}{9}$ of 45 bundles = x bundles, or $9 : 10 : : 45 : x$.

255. The formal expression for the compound proportion is more easily derived from the formal solution by simple proportion. Thus,

3 horses : 5 horses : : 45 bundles : *Ans.* ($\cancel{75}$ bundles).
3 days : 2 days . . $\cancel{75}$ bundles : 2d *Ans.*

If we should multiply the terms together, 75 in the numerator of the ratio would be cancelled by 75 in the denominator. Whence the proportion would become

$$\left.\begin{array}{l}3 \text{ horses} : 5 \text{ horses}\\ 3 \text{ days} : 2 \text{ days}\end{array}\right\} : : 45 : x.^{*}$$

$$\overline{9 : 10 : : 45 : x}$$

Which, reduced, gives

$$9x = 450.$$
$$x = 50.$$

---

* This form may be derived directly from the compound fractional form. Thus,

$$\left[\tfrac{2}{3} \text{ of } \tfrac{5}{3} \text{ of } 45 = x\right] = \left[\frac{2 \times 5}{3 \times 3} \text{ of } 45 = x\right] = \left[\begin{matrix}3 : 5\\ 2 : 3\end{matrix} : : 45 : x\right]$$
$$= [9 : 10 : : 45 : x].$$

Rule for the formal statement and reduction of the compound proportion:

I. *Make the number that is of the same denomination as the answer the third term.*

II. *Make each two of the other numbers that are of the same denomination a factor of the compound ratio, arranging the terms as in simple proportion.*

III. *Reduce the compound proportion to a simple proportion by multiplying the corresponding terms of the first member together.*

IV. *Solve as in simple proportion.*

256. It is sometimes convenient to reduce a problem in compound proportion to one in simple proportion, at the outset. By reasoning that 9 horses will eat in 1 day what 3 horses eat in 3 days, and 10 horses will eat in 1 day what 5 horses eat in 2 days, problem (1) becomes:

If 9 horses eat 45 bundles of oats in 1 day, how many bundles will 10 horses eat in the same time, which gives

$$\tfrac{10}{9} \text{ of } 45 = x;$$

$$\text{or, } 9 \text{ horses} : 10 \text{ horses} :: 45 \text{ bundles} : x \text{ bundles.}$$

$$9x = 450.$$

$$x = 50.$$

To the Teacher. — The formal method of solution of the compound proportion involves so high a degree of abstraction that the average pupil of the grammar school will be injured rather than benefited by an attempt to master it. It is therefore suggested that this method be left for the final review. The simpler methods offer no special difficulty, and this class of problems yields the very best training in consecutive thinking.

Undue emphasis can therefore scarcely be paid to this subject. As the work goes on, the teacher should take the utmost pains to have each and every point thoroughly understood by the pupil.

257. Examples.

1. If 9 men earn $ 99 in 11 days, how much will 6 men earn in 14 days?

2. If 5 men earn $ 11250 in 18 years, how much will 25 men earn in 2 years?

3. If 14 men can mow 84 acres of grass in 6 days, how many acres can 11 men mow in 4 days?

4. If 16 men save $ 84 in 24 days, how many dollars will 36 men save in 16 days?

5. If 4 men can reap 84 acres in 14 days, how many men will reap 75 acres in 15 days?

6. If 12 men can earn $ 30 in 3 days, how many men will earn $ 45 in 8 days?

7. If 70 horses eat 147 bushels of corn in 26 days, in how many days will 39 horses eat 21 bushels?

8. If 70 horses, in 60 days, eat 16 bushels of corn, in what time will 24 horses eat 30 bushels?

9. If 25 men consume 300 bushels of corn in 12 months, how long will 1112 bushels last 139 men?

10. If 19 weavers can weave 57 pieces of cloth, of 25 yards each, in 12 days, working 12 hours a day, how many pieces of 75 yards each can 12 weavers weave in the same time?

11. If for the use of $ 350, for 9 days, I pay $ 5.25, at the same rate what should I pay for the use of $ 500 for 12 days?

12. If a room 124 feet long and 84 feet broad will accommodate 1176 persons, what is the length of a room 72 feet in width which will accommodate 756 persons?

13. If 14 horses eat 120 bushels of corn in 56 days, how long will 94 bushels last 6 horses?

14. If 6 men can dig a trench 34 yards long in 10 days, how many men would be required to dig one 170 yards long in 15 days?

15. If the interest on $624 for 3½ years be $131.04, what will be the interest, at the same rate, on $490 for 4½ years?

16. If 5 horses eat 600 bundles of fodder in 40 days, how many horses will 1800 bundles feed for 60 days?

17. If 96 boards, 15 feet, 6 inches long, and 14 inches wide, will floor a place, how many boards will it take if they are only 11 feet 4 inches long and 9 inches wide?

18. If 8 men can build a wall 82½ yards long in 10 days, how many men will it take to build a wall 247½ yards long in 5 days?

19. If 12 men, in 11 days, build a wall 220 yards long, how many men would be required to build a wall 480 yards long in 18 days?

20. If the freight on 25 tons of coal for 50 miles is $28.75, how much coal should be carried 125 miles for $80.50?

21. If 48 men in 36 days, working 9 hours a day, can build a wall 648 feet long, 6 feet high, and 3 feet thick, how long a wall, that is 8 feet high and 4 feet thick, can 32 men build in 4 days, working 12 hours a day?

22. A cistern 17½ feet in length, 10½ feet in breadth, and 13 feet deep will hold 546 barrels of water. How many barrels of it will it take to fill a cistern 16 feet long, 7 feet broad, and 15 feet deep?

23. If $2030 a year will comfortably maintain a family of 7 people in the United States, what will it cost a family of 9 to live in Canada, in the same style, for 9 months, prices there being $\frac{3}{4}$ of what they are in the United States?

24. If 5 men and 3 boys can pack 1530 crates of peaches in 9 days, working 5 hours a day, how many crates will 8 men and 9 boys pack in 6 days, working 8 hours a day, the rates of working of a man and boy being as 5 to 3?

25. If a merchant, with a capital of $4000, gain $150 in 4 months, how many months will it take him, with a capital of $5000, to gain $300?

26. If a family of 5 persons be supported 3 months for $225, how many persons will be supported 8 months for $1080?

27. If a family of 5 persons be supported 3 months for $225, how much will it take to support a family of 9 persons for 8 months?

28. If a family of 5 persons be supported 3 months for $225, for how many months will $1080 support a family of 9 persons?

29. If 5 men earned $75 in 15 days, how many men will earn $100 in 20 days?

CHAPTER XI.

PERCENTAGE.

258. Per cent is a contraction of the Latin *per centum*, and means by, or in, the hundred. 6 per cent means 6 in the hundred. Thus, 6 per cent of 500 sheep means 6 sheep in each 100 of the 500 sheep. Whence 6 per cent of 500 sheep = 5 times 6 sheep = 30 sheep.

(% = per cent.)

What is

1. 3% of 200 men?
2. 5% of 500 miles?
3. 2% of 400 lb.?
4. 1% of 800 gal.?
5. 7% of 900 oz.?
6. 4% of 1000 in.?
7. 6% of 1200 cu. in.?
8. 8% of 1500 pt.?
9. 9% of 2000 pk.?
10. 10% of 3000 bu.?
11. 12% of $1200?
12. 25% of $800?
13. 2½% of $600?
14. 7½% of $400?
15. 12½% of $800?
16. 30% of $300?
17. 20% of $400?
18. 40% of $200?
19. 50% of $600?
20. 75% of $1200?

259. Per cent may be, and generally is, expressed as a fraction; for 6 per cent, or 6 in the 100, is equal to $\frac{6}{100}$, or .06, of any given expression.

Solve the foregoing by each of the two methods after the manner of the following models:

MODEL 1 (Ex. 1). — 3% of 200 men means 3 men in each 100 of the 200 men; whence 3% of 200 men = 2 times 3 men = 6 men.

Or,

Model 2 (Ex. 1).—3% of 200 men equals $\frac{3}{100}$ of 200 men; $\frac{1}{100}$ of 200 men = 2 men; $\frac{3}{100}$ of 200 men = 3 × 2 men = 6 men; or 6% = .06; .06 of 200 men = .06 times 200 men = 6 men.

Express fractionally, in both the common and the decimal form:

5%	8%	25%	40%	200%	⅕%
2%	9%	12½%	50%	300%	⅔%
3%	10%	7½%	75%	500%	¾%
4%	11%	8⅓%	80%	⅙%	⅚%
6%	12%	21%	90%	⅓%	⅗%
7%	13%	35%	100%	¼%	⅞%

Express fractionally and reduce to their lowest terms:

10%	70%	12½%	35%	4⅙%	125%
20%	80%	37½%	25%	75%	150%
30%	90%	62½%	33⅓%	85%	175%
40%	5%	87½%	66⅔%	250%	½%
50%	2½%	15%	16⅔%	300%	⅔%
60%	7½%	45%	8⅓%	500%	¾%

260. In the solution of problems in percentage, there are five elements that are to be taken into account: the Base; the Rate per Cent, or the Rate; the Percentage; the Sum, or the Amount; the Difference.

These elements are so related to each other that any two being given the others are easily found.

261. The **Base** is the number on which the percentage is reckoned.

262. The **Rate per Cent** is the number that denotes how many units of each 100 (units) of the base are to be taken; the Rate denotes the hundredths of the base that are to be taken. In the example 6% of 500, the rate per cent is 6 (units); the rate is $\frac{6}{100}$, or .06.

263. The **Percentage** is the result obtained by taking any per cent of a number.

NOTE. — The term Percentage is also applied to the general process of reckoning by hundredths.

264. The **Sum** or **Amount** is the base plus the percentage.

265. The **Difference** is the base less the percentage.

265. THE FOUR GENERAL PROBLEMS IN PERCENTAGE.

Problem I.

To find the Percentage, when the Base and Rate are given.

How much is 5% of 800 bushels?

SOLUTION 1.

$$8 \times 5 \text{ bu.} = 40 \text{ bu.}$$

SOLUTION 2.

$$\frac{5}{100} = \frac{1}{20};\ \frac{1}{20} \text{ of } 800 \text{ bu.} = 40 \text{ bu.}$$

SOLUTION 3.

$$\frac{5}{100} \text{ of } 800 \text{ bu.} = \frac{4000 \text{ bu.}}{100} = 40 \text{ bu.}$$

SOLUTION 4.

$$.05 \text{ of } 800 \text{ bu.} = \begin{array}{r} 800 \text{ bu.} \\ .05 \\ \hline 40.00 \text{ bu.} \end{array}$$

By formal proportion.

$$5 \text{ bu.} : 100 \text{ bu.} :: x \text{ bu.} : 800 \text{ bu.}$$
$$100\,x = 4000 \text{ bu.}$$
$$x = 40 \text{ bu.}$$

What is

1. 60% of 320s.?
2. 20% of 740 qt.?
3. 10% of 330 mi.?
4. 25% of 640 lb.?
5. 90% of 120 rd.?
6. 45% of 180 d.?
7. 35% of 360 gal.?
8. 30% of 670 men?
9. 5% of 240 A.?
10. 9% of 200 fr.?
11. 18% of 390 yd.?
12. 33⅓% of $720?
13. 16⅔% of 360 boys?
14. 66⅔% of 240 sheep?
15. 8⅓% of 480 calves?
16. 62½% of 640 horses?
17. 87½% of 560 oranges?
18. 12½% of 480 apples?
19. 37½% of 320 bananas?
20. 2½% of 800 days?
21. A man bought 350 acres of land, and sold 75% of it. How many acres had he left?
22. A man bought a horse at $180, and sold it at a gain of 30%. How many dollars did he gain?
23. A farmer harvested 830 bushels of wheat from one field, and 150% of that amount from another. How many bushels did the second field yield?
24. A merchant, failing in business, compromised his indebtedness at 33⅓%: he owes A $240, B $2072, and C $1500. What does he pay to each?
25. A has $2472 deposited in bank, and wishes to draw out 17½% of it. How much does he write his check for?
26. A cistern which contained 3890 gallons leaked 27½% of it. How much remained?
27. A merchant having 3892 bushels of meal, sold 90% of it. How much remained?
28. Cotton, which was worth 8 cents a pound, has depreciated 37½%. What is it bringing now?

29. A man's income is $1600 a year; he spends 75% of it. How much does he save?

30. A grew 650 bales of cotton, and marketed 90% of them. How many remained unsold?

31. A house and lot was sold for $12500; the purchaser immediately resold it at an advance of $12\frac{1}{2}\%$. How much did he realize by the transaction?

32. A bookkeeper receives a salary of $2700; he spends $33\frac{1}{3}\%$ for groceries, $16\frac{2}{3}\%$ for clothing, $11\frac{1}{9}\%$ for house rent, and 25% for other expenses. How much does he save?

33. A man died, possessed of an estate valued at $180000; he bequeathed 20% of it to a college, 10% of it to a public library, and the remainder equally to his 5 children. How much did each bequest amount to?

34. A's salary is $1200 a year; if he pay 25% of it for board, $16\frac{2}{3}\%$ for clothing, and 30% for other expenses, what are his yearly expenses?

35. A merchant had $8000 to his credit in bank; he drew out $12\frac{1}{2}\%$ of it, then 10% of the remainder, and afterward deposited 25% of what he had withdrawn. How much was he then credited with at the bank?

267. *Problem II.*

To find the Rate, or the Rate per Cent, when the Base and Percentage are given.

40 bushels is what % of 800 bushels?

SOLUTION 1.

To find the Rate per Cent state the question thus:

40 bu. in 800 bu., is how many bu. in each 100 bu.?

If in 8 hundred there are 40 bu., in 1 hundred there are $\frac{1}{8}$ of 40 bu. = 5 bu.

SOLUTION 2.

To find the Rate, or how many hundredths 40 bu. is of 800 bu. Since 1 bu. is $\frac{1}{800}$ of 800 bu., 40 bu. is 40 × $\frac{1}{800}$ or $\frac{40}{800}$ of 800 bu.; $\frac{40}{800} = \frac{1}{20}$; $\frac{1}{20} = \frac{5}{100}$ or .05 or 5%; or reduce $\frac{40}{800}$ directly to a decimal.

By formal proportion:

$$40 \text{ bu.} : x \text{ bu.} :: 800 \text{ bu.} : 100 \text{ bu.}$$

$$800\,x = 4000 \text{ bu.}$$

$$x = 5 \text{ bu.}$$

1. A speculator made $ 4800 on an investment of $ 86400. What was his gain per cent?
2. A school numbers 504 pupils, and on a rainy day 336 pupils answer at roll call. What is the per cent of absence?
3. A man whose salary is $ 900 spends $ 750. What per cent of his salary does he save?
4. A merchant failing in business made an assignment for the benefit of his creditors. His assets amounted to $ 5495, and his liabilities were $ 7850. What per cent of his indebtedness did he pay?
5. A farmer raised 350 bales of cotton, and sold 200 bales. What per cent of his cotton did he hold?
6. From a farm containing 720 acres, 144 acres were sold. What per cent of the farm was sold?
7. The base is 74841, and the percentage 55247. What is the rate per cent?

268. What per cent of

1. 30 is 15?
2. 45 is 15?
3. 45 is 9?
4. 15 is 5?
5. 10 is 5?
6. 10 is 2?
7. 20 is 4?
8. 40 is 8?

9. 80 is 16?
10. 36 is 9?
11. 24 is 8?
12. 48 is 12?
13. 50 gal. is 10 gal.?
14. 72 lb. is 24 lb.?
15. 81 qt. is 27 qt.?
16. 64 rd. is 16 rd.?
17. 36 ft. is 24 ft.?
18. 80 bu. is 2 bu.?
19. 120 mi. is 40 mi.?
20. 96 yd. is 12 yd.?
21. 42 rd. is 12 rd.?
22. 35 in. is 28 in.?
23. 56 ft. is 21 ft.?
24. 72 lb. is 27 lb.?
25. 20 cwt. is 2 qr.?
26. 4 yd. is 2 ft.?
27. 2 ft. is 6 in.?
28. $\frac{3}{4}$ is $\frac{1}{2}$?
29. $\frac{5}{8}$ is $\frac{3}{8}$?
30. $\frac{1}{8}$ is $\frac{1}{4}$?
31. $\frac{5}{6}$ is $\frac{2}{3}$?
32. $7\frac{1}{2}$ is $2\frac{1}{2}$?
33. $12\frac{1}{2}$ is $2\frac{1}{2}$?
34. $37\frac{1}{2}$ is $12\frac{1}{2}$?
35. $62\frac{1}{2}$ is $12\frac{1}{2}$?
36. 900 rd. is 300 rd.?
37. 600 yd. is 200 yd.?
38. 2100 men is 700 men?
39. 9600 mi. is 1600 mi.?
40. 7200 lb. is 800 lb.?
41. 4275 cwt. is 1425 cwt.?
42. 4275 cwt. is 1425 qr.?
43. \$120 is \$72?
44. \$96 is \$84?
45. \$252 is \$168?
46. \$135 is \$75?
47. \$208 is \$156?
48. \$312 is \$104?
49. \$425 is \$272?
50. \$6912 is \$5184?
51. \$8392 is \$1049?
52. \$9475 is \$1137?
53. \$4325 is \$1903?
54. \$9575 is \$3447?
55. \$8425 is \$2359?

269. *Problem III.*

To find the base, when the rate and the percentage are given.

Of what number of bushels is 40 bushels 5%?

SOLUTION 1.

40 bu. is 5 bu. in as many hundred as 5 bu. is contained times in 40 bu., which is 8; hence 40 bu. is 5% of 800 bu.

SOLUTION 2.

If 5% of the base is 40 bu.,
1% of the base is $\frac{1}{5}$ of 40 bu. = 8 bu.
If 1% of the base is 8 bu.,
100% of the base is 100 × 8 bu. = 800 bu.

Or,

$5\% = \frac{5}{100} = \frac{1}{20}$. If $\frac{1}{20}$ of the base is 40 bu.,
$\frac{20}{20}$ of the base is 20 × 40 bu. = 800 bu.

By formal proportion:

$$5 : 100 :: 40 : x.$$
$$5x = 4000.$$
$$x = 800.$$

What is the number of which

1. 10 is 25%?
2. 8 is $12\frac{1}{2}$%?
3. 25 is 5%?
4. 30 is 6%?
5. 40 is 8%?
6. 50 is 10%?
7. 60 is 12%?
8. 45 is 9%?
9. 36 is 6%?
10. 32 is 8%?
11. 24 is 3%?
12. 72 is 6%?
13. 72 is 24%?
14. 96 is 32%?
15. 56 is 7%?
16. 25 is $2\frac{1}{2}$%?
17. $37\frac{1}{2}$ is $12\frac{1}{2}$%?
18. 50 is 50%?
19. 40 is 20%?
20. 60 is 40%?
21. 30 is $\frac{1}{2}$%?
22. 45 is $\frac{3}{4}$%?
23. 72 is $66\frac{2}{3}$%?
24. 60 is $16\frac{2}{3}$%?
25. 25 is $33\frac{1}{3}$%?
26. 50 is $8\frac{1}{3}$%?
27. 75 is $\frac{3}{8}$%?
28. 100 is $\frac{1}{8}$%?
29. $21.60 is 3%?
30. 430 is 86%?
31. 720 is 96%?
32. 720 is $83\frac{1}{3}$%?
33. $1579.50 is $92\frac{6}{7}$%?
34. 1985 is 5%?
35. 1350 is 30%?
36. 1395 is $69\frac{3}{4}$?
37. A man rents his house for $240 a year, and realizes 8% on his investment. What is the value of his property?

38. A merchant spends $3500 for clerks' hire, which is 7% of his income. What is his income?

39. A buys a horse and sells it at a profit of $45, thereby clearing 15% in the transaction. What is the cost of the horse?

40. A merchant sold $2646 worth of goods, and had 65% of his stock left. What was the value of his stock?

41. A man owns 75% of a cotton factory, and sells $66\frac{2}{3}$% of his share for $40000. What is the value of the factory?

270. *Problem IV.*

To find the Base, when the Amount, or Difference, and the Rate are given.

NOTE. — Problem IV. is a modification of Problem III., the amount or difference being reducible to a percentage of the base.

1. What number of bushels increased by 5% of itself equals 840 bushels?

SOLUTION 1. — State the question thus: What number of bushels increased by 5 bu. in each 100 bu. equals 840 bu.? Since each 100 bu. increased by 5 bu. equals 105 bu., the number will be as many hundred bushels as 105 bu. is contained times in 840 bu., which is 8 times; hence the 800 bu. is the required number.

SOLUTION 2. — A number increased by 5% of itself is $\frac{105}{100}$, or $\frac{21}{20}$ of itself; whence

105% of the number = 840 bu.,
1% of the number = 8 bu.,
100% of the number = 800 bu.;

or,

$\frac{21}{20}$ of the number = 840 bu.,
$\frac{1}{20}$ of the number = 40 bu.,
$\frac{20}{20}$ of the number = 800 bu.

By formal proportion:

$$105 : 100 :: 840 : x.$$
$$105x = 84000.$$
$$x = 800.$$

2. What number of bushels decreased by 5% of itself equals 760 bushels?

Solution 1. — State the question thus: What number of bushels decreased by 5 bu. in each 100 bu. equals 760 bu.? Since each 100 bushels decreased by 5 bu. equals 95 bu., the number will express as many hundred bushels as 95 bu. is contained times in 760 bu., which is 8; hence 800 bu. is the required number.

Solution 2. — A number decreased by 5% of itself equals 95%, or $\frac{19}{20}$ of itself; whence

$$\begin{aligned} 95\% \text{ of the number} &= 760 \text{ bu.,} \\ 1\% \text{ of the number} &= 8 \text{ bu.,} \\ 100\% \text{ of the number} &= 800 \text{ bu.;} \end{aligned}$$

or,

$$\begin{aligned} \tfrac{19}{20} \text{ of the number} &= 760 \text{ bu.,} \\ \tfrac{1}{20} \text{ of the number} &= 8 \text{ bu.,} \\ \tfrac{20}{20} \text{ of the number} &= 800 \text{ bu.} \end{aligned}$$

By formal proportion.

$$95 : 100 : : 760 : x.$$
$$95x = 76000.$$
$$x = 800.$$

What number increased by

3. 20% of itself = 30?
4. 10% of itself = 55?
5. 5% of itself = 42?
6. 25% of itself = 60?
7. 50% of itself = 15?
8. 75% of itself = 21?
9. 100% of itself = 12?
10. 150% of itself = 25?
11. 4% of itself = 52?
12. 2% of itself = 204?
13. $33\frac{1}{3}\%$ of itself = 64?
14. $16\frac{2}{3}\%$ of itself = 56?
15. $8\frac{1}{3}\%$ of itself = 26?
16. $6\frac{1}{4}\%$ of itself = 68?
17. $37\frac{1}{2}\%$ of itself = 88?
18. $62\frac{1}{2}\%$ of itself = 65?
19. 18% of itself = 147.50?
20. 25% of itself = 44.45?
21. 12% of itself = 560?
22. $33\frac{1}{3}\%$ of itself = 1000?
23. $37\frac{1}{2}\%$ of itself = 5.50?
24. 20% of itself = 158.94?
25. $16\frac{2}{3}\%$ of itself = 196.42?
26. 15% of itself = 231.25?

What number decreased by

27. 20% of itself = 40?
28. 10% of itself = 45?
29. 5% of itself = 38?
30. 25% of itself = 60?
31. 50% of itself = 15?
32. 75% of itself = 21?
33. 100% of itself = 0?
34. 30% of itself = 42?
35. 4% of itself = 72?
36. 2% of itself = 49?
37. $33\frac{1}{3}$% of itself = 48?
38. $16\frac{2}{3}$% of itself = 65?
39. $8\frac{1}{3}$% of itself = 99?
40. $6\frac{1}{4}$% of itself = 90?
41. $37\frac{1}{6}$% of itself = 50?
42. $62\frac{1}{6}$% of itself = 36?
43. 20% of itself = 35.50?
44. $12\frac{1}{6}$% of itself = 63.56?
45. 15% of itself = 63.75?
46. 10% of itself = 787.50?
47. 16% of itself = 841.68?
48. $87\frac{1}{6}$% of itself = $632.37\frac{1}{6}$?
49. $37\frac{1}{6}$% of itself = 18.90?
50. $16\frac{2}{3}$% of itself = 56.70?
51. 15% of itself = 7.82?

52. A man sold a horse for $ 234, which was an advance of 30% on the cost. What was the cost?

53. A merchant issues his check on the Merchants and Planters' Bank of Griffin, Ga., for $ 2904.60, which is $17\frac{1}{2}$% more than he has on deposit. By what amount does he overdraw his deposit?

54. A merchant sold his interest in a stock company for $ 26464, which was an advance of $3\frac{3}{8}$% on what he paid for it. What did it cost him?

55. A's income has been increased $13\frac{1}{2}$%, and it is now $ 1760. Find his former salary.

56. The population of a certain town is 25804, which is $15\frac{2}{3}$% more than it was at the time of the census. What was the population then?

57. A bankrupt compromised with his creditors for $ 10000, which was $58\frac{1}{3}$% off. What was the amount of his indebtedness?

271. EXAMPLES IN PERCENTAGE.

Oral Exercises.

A per cent of a number is so many hundredths of it; 5 per cent is $\frac{5}{100}$; 25 per cent is $\frac{25}{100}$; 50 per cent is $\frac{50}{100}$; 100 per cent is $\frac{100}{100}$.

272. 1. What part is 5 per cent?

EXPLANATION. — 5 per cent = $\frac{5}{100}$; $\frac{5}{100}$ reduced to its lowest terms is $\frac{1}{20}$.

2. What part is 10 per cent? 20? 30? 40? 50? 60? 70? 80?
3. What part is 25 per cent? 50? 75? 12½?
4. What part is 2 per cent? 6? 8? 4?
5. What part is 7 per cent? 14? 28?
6. What part is 35 per cent? 37½? 62½?
7. What part is 2½ per cent? 7½?
8. What part is 33⅓ per cent? 66⅔? 16⅔? 8⅓?
9. What part is 12½ per cent? 6¼? ⅜? 87½?

273. 1. How many per cent in 1? 2? 3?

2. How many per cent in ½? ⅛? ¼?
3. How many per cent in ⅓? ⅔? ⅜?
4. How many per cent in ⅙? ⅕? ⅗?
5. How many per cent in ⅕? $\frac{1}{10}$? $\frac{7}{10}$?
6. How many per cent in ¼? ¾? ⅘?
7. How many per cent in ⅙? ⅚? $\frac{1}{10}$?
8. How many per cent in $\frac{3}{10}$? $\frac{5}{10}$? $\frac{9}{10}$?
9. How many per cent in $\frac{1}{12}$? $\frac{5}{12}$? $\frac{1}{24}$?

10. How many per cent in $\frac{1}{20}$? $\frac{3}{20}$? $\frac{15}{20}$?

11. How many per cent in $\frac{1}{15}$? $\frac{1}{16}$? $\frac{1}{8}$?

12. How many per cent in $\frac{1}{25}$? $\frac{3}{25}$? $\frac{7}{25}$?

13. How many per cent in $\frac{1}{50}$? $\frac{1}{40}$? $\frac{3}{50}$?

14. How many per cent in $\frac{2}{3}$? $\frac{5}{6}$? $\frac{7}{8}$?

274. 1. 3 is what per cent of 4?

SOLUTION.—3 is $\frac{3}{4}$ of 4; $\frac{3}{4} = \frac{75}{100}$ or 75 per cent.

2. 4 is what per cent of 8? 12? 16? 20? 40?

3. 5 is what per cent of 10? 15? 20? 25? 40?

4. 6 is what per cent of 12? 18? 24? 30? 36 ?

5. 8 is what per cent of 16? 24? 32? 40? 48?

6. 20 dollars are what per cent of 60 dollars?

7. John bought an orange for 2 cents, and sold it for 3 cents. What per cent did he gain?

SOLUTION.—He gained 1 cent on 2 cents = $\frac{1}{2} = \frac{50}{100}$, or 50 per cent.

8. A merchant bought cloth at 3 dollars per yard, and sold it at 4 dollars per yard. What per cent did he gain?

9. In the last example the profit was what part of the cost? of the selling price? The cost was what part of the selling price? The selling price what part of the cost?

10. Bought a melon for 5 cents, and sold it for 4 cents; What per cent did I lose?

11. In the last example, the loss was what part of the cost? of the selling price? The cost was what part of the selling price? The selling price what part of the cost?

12. A watch was bought for 12 dollars, and sold for 15 dollars. What was the gain per cent?

13. Bought goods for 25 dollars, and sold them for 50 dollars. What was the gain per cent?

14. Bought for $ 30 and sold for $ 45. Gained what per cent?

15. Bought for $ 72 and sold for $ 63. Lost what per cent?

16. Bought for $ 90 and sold for $ 99. Gained what per cent?

17. Bought for 64 dollars and sold for 72 dollars. Gained what per cent?

18. Bought for $ 48 and sold for $ 42. Lost what per cent?

19. Bought for $ 44 and sold for $ 66. Gained what per cent?

20. Bought for $ 100 and sold for $ 106. Gained what per cent?

21. Bought for $ 50 and sold for $ 46. Lost what per cent?

22. Bought for $ 45 and sold for $ 54. Gained what per cent?

23. Bought for $ 54 and sold for $ 45. Lost what per cent?

275. **1.** I sold a horse for $ 81, and gained $12\frac{1}{2}$ per cent. What was the cost?

Solution. — $12\frac{1}{2}$ per cent is $\frac{1}{8}$. Since the cost plus the profit gives the selling price, $\frac{8}{8} + \frac{1}{8} = \frac{9}{8}$; $\frac{9}{8}$ cost $= 81$; $\frac{1}{8} = 9$; $\frac{8}{8} = 72$.

2. Sold a watch for $ 63, and lost $12\frac{1}{2}$ per cent. What was the cost?

3. Sold an article for $ 39, and gained 30 per cent. What was its cost?

4. Sold an article for 28 cents, and gained 40 per cent. What was the cost?

5. Sold for 63 cents, and lost 10 per cent. What was the cost?

6. Sold for $ 14, and gained 75 per cent. What was the cost?

7. Sold for $ 28, and lost $12\frac{1}{2}$ per cent. What was the cost?

8. Sold for $ 25, and gained 25 per cent. What was the cost?

9. Sold for $ 12, and lost 60 per cent. What was the cost?

10. Sold for $ 18, which was 20 per cent more than the cost. What was the cost?

11. Gained $ 8 on a sale, which was 4 per cent of the cost. What was the cost?

12. When the selling price is $\frac{3}{2}$ of the cost, what is the gain per cent?

13. When the selling price is $\frac{3}{4}$ of the cost, what is the loss per cent?

14. When the cost is $\frac{3}{5}$ of the selling price, what is the gain per cent?

15. When the cost is $\frac{4}{3}$ of the selling price, what is the loss per cent?

16. When the profit is $\frac{1}{5}$ of the selling price, what is the gain per cent?

17. A boy collects $ 350, and receives 5 per cent commission on what he collects. How many dollars does he get?

18. What will it cost to collect a note of $ 650, at 10 per cent commission?

19. The sum collected was $ 1200; commission 5 per cent. What was the cost of collecting?

20. The sum collected was $ 1000; commission 12½ per cent. What was the cost of collecting?

21. A broker sells $ 800 worth of Georgia R. R. stock, and charges ¾ per cent for selling. How much does he receive?

22. What will be the cost of negotiating a bill of exchange for $ 1500, at ⅗ per cent?

23. What will it cost to insure a house for $ 2500, at 1⅕ per cent premium?

24. What will be the premium for insuring a store valued at $ 4000, at 1¾ per cent?

25. Sum insured was $ 2000; rate 3½ per cent. What was the premium?

26. Sum insured was $ 600; rate ⅛ per cent. What was the premium?

27. At 2 per cent, what amount of insurance can be had for $ 6? $ 8? $ 24?

28. At 6 per cent, what amount of insurance can be had for $ 12? $ 18? $ 36?

29. At 5 per cent, what amount of insurance can be had for $ 20? $ 25? $ 30?

30. At ½ per cent, what amount of insurance can be had for $ 6? $ 8? $ 9?

31. At 2½ per cent, what amount of insurance can be had for $ 25? $ 125?

32. At ¾ per cent, what amount of insurance can be had for $ 9? $ 15? $ 18?

276. 1. What is the interest of $200 for a year, at 5 per cent?

SOLUTION. — 5 per cent, or $\frac{5}{100} = \frac{1}{20}$; $\frac{1}{20}$ of $200 = $10.

2. What is the interest of $240 for a year, at $12\frac{1}{2}$ per cent?

3. What is the interest of $25 for 3 years, at 4 per cent?

4. What is the interest of $80 for 2 years, at 5 per cent?

5. What is the interest of $75 for 4 years, at 9 per cent?

6. What is the interest of $400 for 5 years, at 3 per cent?

7. What is the interest of $80 for 2 years, at $7\frac{1}{2}$ per cent?

8. What is the interest of $60 for 5 years, at $6\frac{1}{4}$ per cent?

9. What is the interest of $75 for 6 months, at 8 per cent?

SOLUTION. — 6 months is $\frac{1}{2}$ year; the interest for 6 months is $\frac{1}{2}$ of 8 per cent = 4 per cent; $\frac{4}{100} = \frac{1}{25}$; $\frac{1}{25}$ of $75 = $3.

10. What is the interest of $50 for 9 months at 8%?

11. Find the interest of $300 for 1 yr. 6 mo. at 6%.

12. When the interest for 2 yr. is $\frac{1}{6}$ of the principal, what is the Rate?

13. When the interest for 2 yr. 6 mo. is $\frac{1}{4}$ of the principal, what is the Rate?

14. When the interest for 3 yr. 8 mo. is 33% of the principal, what is the Rate?

15. What principal will amount to $44 in a year at 10%?

16. What principal will amount to $92 in 2 yr. 6 mo. at 6%?

17. What principal will yield $18 interest in 2 yr. at 8%?

18. What principal will yield $72 interest in 2 yr. 3 mo. at 8%?

19. What principal will yield $96 interest in 2 yr. 8 mo. at 9%?

20. When the interest, at 6%, equals 24% of the principal, what is the time?

21. When the interest, at 8%, equals ¼ of the principal, what is the time?

What is the amount of a bill of:

1. $100 with 20% and 10% off?

Note. — Commercial Discount is a per cent off of anything, without regard to time.

Solution. — 20% of $100 = $20; $100 − $20 = $80; 10% of $80 = $8; $80 − $8 = $72?

2. $80 with 25% off?
3. $75 with 4% off?
4. $36 with 25% off?
5. $42 with 16⅔% off?
6. $72 with 12½% off?
7. $48 with 33⅓% off?
8. $60 with 20% and 25% off?
9. $150 with 33⅓% and 50% off?

The Four Problems.

Note. — It is suggested that sections 277–287 be omitted until the final review.

277. *Summary and Generalization.*

Problem I. — Given Base and Rate Per Cent, to find Percentage.

Problem II. — Given Base and Percentage, to find the Rate.

Problem III. — Given Percentage and Rate, to find the Base.

Problem IV. — Given Amount or Difference and Rate, to find the Base.

278. *Special Example of each Problem.*

Problem I. — What is 5% of 800 bu.?

Problem II. — What % of 800 bu. is 40 bu.?

Problem III. — Of what number of bushels is 40 bu. 5%?

Problem IV. — (*a*) What number of bushels increased by 5% of itself equals 840 bu.?

Problem IV. — (*b*) What number of bushels decreased by 5% of itself equals 760 bu.?

279. *Restatement of Special Example of each Problem.*

Problem I. — 5 bu. in each 100 bu. is how many in 800 bu.?

Problem II. — 40 bu. in 800 bu. is how many bushels in each 100 bu.?

Problem III. — 40 bu. is 5 bu. in each of how many 100 bu.?

Problem IV. — (*a*) 840 bu. is 105 bu. in each of how many 100 bu.?

Problem IV. — (*b*) 760 bu. is 95 bu. in each of how many 100 bu.?

280. *Solution of each Example by Analysis or Primary Proportion.*

Problem I. — 5 bu. in each 100 bu. is in 800 bu., 8 times 5 bu. = 40 bu.

Problem II. — 40 bu. in 800 bu. is in 100 bu., $\frac{1}{8}$ times 40 bu. = 5 bu.

Problem III. — 40 bu. is 5 bu. in each of as many 100 bu. as 5 bu. is contained times in 40 bu., which is 8 (100) bu.

Problem IV. — (*a*) 840 bu. is 105 bu. in each of as many 100 bu. as 105 bu. is contained times in 840 bu., which is 8 (100) bu.

Problem IV. — (*b*) 760 bu. is 95 bu. in each of as many 100 bu. as 95 bu. is contained times in 760 bu., which is 8 (100) bu.

281. *Solution of each Example by Formal Proportion.*

Problem I. — 5 bu. : 100 bu. : : x bu. : 800 bu.; whence $x = 40$ bu.

Problem II. — 40 bu. : 800 bu. : : x bu. : 100 bu.; whence $x = 5$ bu. (in each 100 bu.).

Problem III. — 40 bu. : x bu. : : 5 bu. : 100 bu.; whence $x = 800$ bu.

Problem IV. — (*a*) 840 bu. : x bu. : : 105 bu. : 100 bu.; whence $x = 800$ bu.

Problem IV. — (*b*) 760 bu. : x bu. : : 95 bu. : 100 bu.; whence $x = 800$ bu.

Note. — If A, who is reaping a field of wheat, gets 8 bu. in each 100 bu. reaped, as his wages, what part of the harvest will fall to his share? 8 bu. in each 100 bu. = $\frac{8}{100}$ of each 100 bu. = $\frac{8}{100}$ of the number of bushels harvested.

It has been found more convenient to regard per cent as meaning so many hundredths of an expression, and not so many units in each 100 units. This requires the per cent to be expressed in the fractional form, and usually the decimal. From this conception of per cent as a fraction, we have the following:

282. *Solution of each Example by Analysis or Primary Proportion.*

Problem I.—.01, or $\frac{1}{100}$ of 800 bu. = 8 bu.; .05, or $\frac{5}{100}$ of 800 bu. = 5 × 8 bu. = 40 bu.; or .05 = $\frac{1}{20}$; $\frac{1}{20}$ of 800 bu. = 40 bu.

Problem II.—1 bu. is $\frac{1}{800}$ of 800 bu.; 40 bu. is 40 × $\frac{1}{800}$ of 800 bu. = $\frac{40}{800}$ of 800 bu. = $\frac{1}{20}$ of 800 bu. = .05 of 800.

Problem III.—.05, or $\frac{5}{100}x = 40$ bu.; .01, or $\frac{1}{100}x = 8$ bu; 1.00 or $\frac{100}{100}x = 800$ bu.: or $\frac{1}{20}x = 40$ bu.; $\frac{20}{20}x = 800$ bu.

Problem IV.—(*a*) 1.05, or $\frac{105}{100}x = 840$ bu.; .01, or $\frac{1}{100}x = 8$ bu.; 1.00, or $\frac{100}{100}x = 800$ bu.: or $\frac{21}{20}x = 840$ bu.; $\frac{1}{20}x = 40$ bu.; $\frac{20}{20}x = 800$ bu.

Problem IV.—(*b*) .95, or $\frac{95}{100}x = 760$ bu.; .01, or $\frac{1}{100}x = 8$ bu.; 1.00, or $\frac{100}{100}x = 800$ bu.: or $\frac{19}{20}x = 760$ bu.; $\frac{1}{20}x = 40$ bu.; $\frac{20}{20}x = 800$ bu.

283. *Solution of each Example by Fractional Proportion.*

Problem I.—.05, or $\frac{5}{100}$, of 800 bu. = .05 × 800 bu. = 40 bu.

Problem II.—40 bu. is $\frac{40}{800}$ of 800 bu.; $\frac{40}{800} = \frac{40.00}{800} = .05$.

Problem III.—.05, or $\frac{5}{100}x = 40$ bu.; 1.00, or $\frac{100}{100}x = \frac{100}{5}$ of 40 bu. = 800 bu.

Problem IV.—(*a*) 1.05, or $\frac{105}{100}x = 840$ bu.; 1.00, or $\frac{100}{100}x = \frac{100}{105}$ of 840 bu. = 800 bu.

Problem IV.—(*b*) .95, or $\frac{95}{100}x = 760$ bu.; 1.00, or $\frac{100}{100}x = \frac{100}{95}$ of 760 bu. = 800 bu.

284. *Solution of each Example by Formal Proportion.*

PROBLEM I. — $.05 : 1.00 :: x : 800$; whence $x = 40$.

PROBLEM II. — $40 : 800 :: .0x : 1.00$; whence $\frac{x}{100} = .05$.

PROBLEM III. — $40 : x :: .05 : 1.00$; whence $x = 800$.

PROBLEM IV. — (*a*) $840 : x :: 1.05 : 1.00$; whence $x = 800$.

PROBLEM IV. — (*b*) $760 : x :: 95 : 1.00$; whence $x = 800$.

Multiplying the means and extremes of the foregoing proportions together they reduce to the following equations:

285. PROPORTION I. $= 1.00x = .05 \times 800$; whence $x = .05 \times 800 = 40$.

PROPORTION II. $= 800 \times \frac{x}{100} = 40$; whence $\frac{x}{100} = \frac{40}{800} = .50$.

PROPORTION III. $= .05x = 40$; whence $x = 40 \div .05 = 800$.

PROPORTION IV. (*a*) $= 1.05x = 840$; whence $x = 840 \div 1.05 = 800$.

PROPORTION IV. (*b*) $= 95x = 760$; whence $x = 760 \div .95 = 800$.

286. In Proportion I., $x =$ Percentage; in II., $\frac{x}{100} =$ Rate; in III., $x =$ Base; in IV. (*a*) and IV. (*b*), $x =$ Base; whence it may be seen by inspection that

(I.) Percentage = Base × Rate,
(II.) Rate = Percentage ÷ Base,
(III.) Base = Percentage ÷ Rate,
(IV. (*a*)) Base = Amount ÷ (1 + Rate),
(IV. (*b*)) Base = Difference ÷ (1 − Rate).

By definition, Amount = Base + Percentage.
By definition, Difference = Base − Percentage.

Letting B stand for base, P for Percentage, R for Rate, A for Amount, and D for Difference, we have the following abbreviated

FORMULAS.

$$P = B \times R,$$

$$R = P \div B, \text{ or } \frac{P}{B},$$

$$B = P \div R, \text{ or } \frac{P}{R},$$

$$B = A \div (1 + R), \text{ or } \frac{A}{1 + R},$$

$$B = D \div (1 - R), \text{ or } \frac{D}{1 - R},$$

$$A = B + P,$$

$$D = B - P.$$

287. EXAMPLES FOR REVIEW.

1. What is $16\frac{2}{3}\%$ of 420?
2. What is $87\frac{1}{2}\%$ of 720?
3. What is $62\frac{1}{2}\%$ of 480?
4. 20 is what % of 80?
5. 80 is what % of 20?
6. 30 is what % of 90?
7. 90 is what % of 30?
8. 30% of what number is 30?
9. $12\frac{1}{2}\%$ of what number is 25?
10. $37\frac{1}{2}\%$ of what number is 72?
11. $62\frac{1}{2}\%$ of 48 is what % of 50?
12. $66\frac{2}{3}\%$ of 36 is what % of 72?
13. 40% of 30 is what % of 84?
14. 50% of 75 is 25% of what number?

15. $6\frac{1}{4}\%$ of 64 is 5% of what number?

16. 25 is $33\frac{1}{3}\%$ more than what number?

17. 36 is 20% more than what number?

18. 72 is 20% more than what number?

19. 81 is $12\frac{1}{2}\%$ more than what number?

20. 110 is 10% more than what number?

21. 45 is $37\frac{1}{2}\%$ less than what number?

22. 64 is $33\frac{1}{3}\%$ less than what number?

23. 50 is $16\frac{2}{3}\%$ less than what number?

24. 20% of 50% is what % of 40% of a number?

25. 25% of 40% of 50 is what % of 30?

26. 60% of 45 is what % of 90% of 36?

27. 30% of 20% is what % of 12% of a number?

28. What % of a number is $\frac{1}{2}$ of it? $\frac{1}{3}$? $\frac{5}{6}$? $\frac{3}{12}$?

29. What % of a number is $\frac{3}{4}$ of it? $\frac{5}{8}$? $\frac{7}{8}$? $\frac{5}{6}$?

30. A man having $75 spent $25 for a suit of clothes. What % of his money had he left?

31. A man's monthly salary is $72 and his board costs him $30 per month. What % of his salary does he spend for board?

32. A teacher receives $1200 a year, and his expenses are $66\frac{2}{3}\%$ of his income. What are his expenses?

33. A man, having $840, invested $735 in a vacant lot. What % of his money had he left?

34. A merchant paid a sight draft of $960, which took $85\frac{5}{7}\%$ of all the cash he had on hand. How much cash remained?

35. A ranchman lost 16⅔% of his sheep by distemper; he sold 60% of the remainder, and had 180 sheep left. What % of his flock remained?

36. A man invested $2250 in a house and lot, which was 62½% of what he was worth. What was he worth?

37. A man owning 75% of a factory sold 66⅔% of his share for $8720. What was the value of the factory?

38. A merchant failing in business found his indebtedness to be $7895.75, and his assets $5692.3145. What % of his debts did he pay?

39. A merchant borrowed $2275 for a year, and paid $955.50 for its use. What % did he pay?

40. What must a bookkeeper's monthly salary be that 6¼% of it shall be $87.75?

41. A merchant deposited in bank $1750.65, which was 20% more than he had taken in that day. What were that day's receipts?

42. A farmer plants 27% of his arable land in corn, 33⅓% of the remainder in cotton, and sows 50% of what still remains, which is 85 A., 40 sq. rd., 9 sq. yd., 6 sq. ft., and 17.28 sq. in., in grain for pasturage. How much land has he in cultivation? Ans. 275 acres.

43. The average number of pupils in attendance in a certain school is 372, which is 93% of the number enrolled. What is the enrollment?

44. A merchant sold 500 sacks of flour, which was 20% of the number of sacks in stock. How many sacks were left?

45. A house and lot was sold for $12,784, which was 6¼% more than it cost. What was the cost?

46. The population of a certain city was, in 1890, 140,448, which was 15½% greater than its population in 1880. What was its population in 1880?

47. An orchard contains 3840 trees, of which 15% are plum, 45% peach, 12½% apple, and the rest pear. How many trees of each kind does the orchard contain?

48. At a forced sale, a house and lot was sold for $ 9124.50, which was 16⅔% less than its real value. If the property had been sold for $ 13,095.4824, how much % above its real value would it have brought?

49. A merchant's loss in the year 1894 was $ 2160, which was 125% of his profit for the year before. What was his profit for the year 1893?

50. The assessed value of the property of a town is $ 3,240,000, and the yearly expense of running its public schools is $ 8050. What is the rate of taxation for the support of the schools?

51. Mr. Doe's state tax is $ 64.50, and the rate is 4 mills on the dollar. What is the assessed value of his property?

52. A man owning ⅔ of the stock of a mining company sold 16⅔% of his share at 12½% above par for $ 7200. What was the par value of the entire stock of the company?

CHAPTER XII.

APPLICATIONS OF PERCENTAGE.

PROFIT AND LOSS.

288. **Cost** is the price paid for anything.

Selling Price is the sum received for anything.

289. Where anything is sold for more than its cost, the Excess is **Profit**; when sold for less than its cost, the Deficiency is **Loss.**

The gain or loss is reckoned at some % of the cost.

290. When abstract percentage is applied to Profit and Loss —

(1) The Base becomes the Cost.

(2) The Rate becomes the Rate of Profit or Loss.

(3) The Percentage becomes the Profit or Loss.

(4) The Amount becomes the Cost plus the Profit, or the Selling Price.

(5) The Difference becomes the Cost less the Loss, or the Selling Price.

291. Whence the following

FORMULAS.

Cost × Rate = Profit or Loss.
Cost + Profit = Selling Price.
Cost − Loss = Selling Price.
Profit or Loss ÷ Rate % = Cost.
Profit or Loss ÷ Cost = Rate.
The Selling Price (amount) ÷ (1 + rate) = Cost.
The Selling Price (difference) ÷ (1 − rate) = Cost.

292. EXAMPLES.

1. Bought 25 gallons of syrup at 65 cents a gallon, and sold all of it for \$18.33$\frac{1}{8}$. What was the gain %?

2. A merchant bought 2765 bushels of corn, for which he paid \$1520.75; but, finding it damaged, he was forced to sell it at a sacrifice of 10%. What did he sell it for per bushel?

3. A farmer sells 250 bushels of oats at 50 cents a bushel. The purchaser sells it at a profit of 10%. How much does the latter get for the oats?

4. A trader bought a cow for \$65.50, and sold her at an advance of 30%. How much did he gain?

5. A butcher buys 300 lb. of beef at 7$\frac{1}{2}$ cents a pound, and wishes to sell it at an advance of 20%. How much must he ask for it per pound?

6. Bought cloth at 45 cts. a yard, and sold it at 60 cts. a yard. What % was gained?

7. Bought silk at 175 cts. per yard, and sold it at 225 cts. per yard. What was the gain per cent?

8. A jobber bought \$3840 worth of dry goods, and sold the lot at a profit of \$768. What was his gain per cent?

9. A factor sold a shipment of cotton for \$4905, which was 5% below the cost. Required the cost.

10. A merchant buys shoes at \$3.75 a pair, and sells them at \$4.76$\frac{1}{4}$ a pair. What is his gain per cent?

11. A trader sold two horses for \$150 each. On the one he gained 25% and on the other he lost 25%. Did he gain or lose on the two, and how much?

COMMISSION.

293. **Commission** is the sum paid an agent, or commission merchant, for transacting business for another, and is reckoned at a certain % on the money received from a sale, or invested by the agent.

294. When abstract percentage is applied to the commission business —

The Base becomes the Sum invested or received.

The Percentage becomes the Commission.

The Rate becomes the Rate of Commission.

The Amount becomes the Sum invested plus the Commission.

The Difference becomes the Sum received less the Commission.

295. Whence the following

FORMULAS.

Sum invested or received × Rate = Commission.
Sum invested + Commission = Amount.
Sum received − Commission = Net Proceeds.
Commission ÷ Rate = Sum invested or received.
Commission ÷ Sum invested or received = Rate.
Amount ÷ (1 + Rate) = Sum invested.
Net Proceeds ÷ (1 − Rate) = Sum received.

296. EXAMPLES.

1. My agent sells goods to the amount of $ 2562.50. What is his commission at 2½%.
2. A real estate agent charging 5% commission remits to the owner of the property as net proceeds of its sale $ 12,825. What price did the property bring, and what was his commission ?

3. What will be an agent's commission for selling a house worth $ 6472 at 3¾% commission?

4. What will be an agent's commission for selling $ 1860 worth of goods at 2⅞% commission?

5. A lawyer collects $ 535, at 2½% commission. After deducting his commission, how much is remitted to his employer?

6. A sends his agent $ 1454.25 to invest in merchandise after deducting his commission of 5%. What sum will be expended in merchandise, and what will be the agent's commission?

7. A commission merchant receives $ 1261.70, with instructions to invest that amount in wheat after deducting his commission of 3⅛%. How much does he invest in wheat?

8. What amount must be collected by a lawyer that he may retain his fee of 5% and remit to his client $ 1323.35?

STOCKS.

297. Stock is the capital invested in any corporate business; that is, in a business conducted by a company established by law and authorized to act as an individual.

298. A **Share** is one of the equal parts into which the stock of a company is divided; it is usually $ 100.

299. The market value of stock is the sum which it will bring.

300. Stock is at Par when it will bring its nominal or face value; above Par, or at a Premium, when it will bring more than its nominal value; and below Par, or at a discount, when it brings less than its nominal value.

301. A **Broker** is a person who buys or sells stocks for himself or for another, and Brokerage is the fee charged by the broker for buying or selling stock. It is reckoned at a certain per cent of the par value of the stock.

302. In the application of abstract percentage to the business of a broker—

The Base becomes the Par Value of Stock.

The Percentage becomes the Brokerage.

The Rate becomes the Rate of Brokerage.

Amount becomes the Sum invested plus the Brokerage.

The Difference becomes the Sum received less the Brokerage.

303. Whence the following

FORMULAS.

Par Value × Rate = Brokerage.
Sum invested + Brokerage = Amount.
Sum received − Brokerage = Net Proceeds.
Brokerage ÷ Rate = Par Value.
Brokerage ÷ Par Value = Rate.

304. 1. What is the cost of 300 shares of Georgia R. R. stock at 112, brokerage being $\frac{1}{4}\%$?

SOLUTION.—Each share will cost $112 plus $\frac{1}{4}\%$ of its par value, or 25 cents = $112.25; and 300 shares will cost 300 × $112.28 = $33684.

2. Find the cost of 36 shares of the Merchants and Planters' Bank of Griffin, Ga., at $115\frac{1}{2}$, brokerage being $\frac{1}{8}\%$?

3. How much will 25 shares of the Kincaid Manufacturing Company stock cost at 140, brokerage being $\frac{1}{8}\%$?

4. Find the cost of 132 shares of the Standard Oil Company stock at $97\frac{3}{4}$, brokerage being $\frac{1}{8}\%$.

5. What will be the cost of 59 shares of the Southern Express Company stock at 102, brokerage being $\frac{1}{8}\%$?

6. Bought 120 shares of bank stock at 93, and sold them for 105, brokerage being $\frac{1}{8}\%$ in each case. What was the gain?

7. Find the cost of 105 shares of the Postal Telegraph Company stock at a premium of $5\frac{3}{4}\%$.

8. What will 75 shares of a Life Insurance Company stock come to at $9\frac{1}{4}\%$ discount, brokerage being $\frac{1}{8}\%$?

9. How many shares of bank stock can be bought for $11687 at 6% discount, brokerage being $\frac{1}{4}\%$?

10. A merchant receives 9% dividend on stock bought at 25% below par. What rate of interest does he get on his investment?

11. How much may be paid for stock that pays a dividend of 12% to realize 6% on the investment?

12. What rate of interest will a man realize by investing in 6% stock at 108?

13. Which is the better investment, 6% stock at 80, or 7% stock at 90?

14. What must I pay for 5% stock that I may realize 8% on my investment?

15. What sum must be invested in 6% stock at 94 to yield an income of $640?

16. What income will an investment of $11931 in 6% stock at 97 yield?

17. What must be paid for a 6% railroad stock that 9% may be realized on the investment?

INSURANCE.

305. Insurance is guaranty by one party of indemnity to another party for loss or damage.

306. The Policy is the written agreement between the insurer, or underwriter, and the insured; and the Premium is the sum paid for the insurance.

307. 1. What is the annual premium on a policy insuring a house for $ 2500, the rate being 2½% ?

2. A ship's cargo was insured for $ 82500 at 2¾%. What was the premium?

3. A cotton factory worth $ 125000 was insured for ⅗ of its value at 1½%. Find the premium.

4. The premium paid for the insurance of a house was $ 130.50, the rate being 1½%. What is the value of the house?

5. A merchant paid $ 55.50 for an insurance of $ 9250 on his stock of goods. Find the rate of insurance.

6. An insurance company charges $ 2.85 for insuring furniture for $ 500. What is the rate?

CHAPTER XIII.

INTEREST AND DISCOUNT.

I. SIMPLE INTEREST.

308. **Interest** is the sum paid for the use of money.

309. The **Principal** is the sum on which interest is paid.

310. The **Rate of Interest** is the number of hundredths of the principal paid for its use for one year.

311. The **Amount** is the principal plus the interest.

312. In the application of abstract percentage to interest —

The Base becomes the Principal.
The Percentage becomes the Interest.
The Rate becomes the Rate of Interest.
The Amount becomes the Principal plus Interest.

313. Whence the following

FORMULAS.

Principal × **Rate** = Interest.
Principal + **Interest** = Amount.
Interest ÷ **Rate** = Principal.
Interest ÷ **Principal** = Rate.
Amount ÷ (1 + **Rate**) = **Principal.**

NOTE. — In computing interest 12 months is accounted a year, and 30 days a month.

314. 1. What is the interest of $ 375.50 for 5 years at 7%?

SOLUTION.

$375.50 = Principal.
.07 = Rate for 1 year.
$26.2850 = Interest for 1 year.
5
$131.4250 = Interest for 5 years.

Or

$375.50 = Principal.
.35 = Rate for 5 years.
187750
112650
$131.4250 = Interest for 5 years.

Find the interest of

2. $ 640.85 for 3 years at 6%.
3. $ 472.25 for 2 years at 5%.
4. $ 783.20 for 4 years at 8%.
5. $ 849.80 for 5 years at 6%.
6. $ 963.30 for 8 years at 4%.
7. $ 643.75 for 3 years at 7%.
8. $ 897.90 for 6 years at 5%.
9. $ 964.46 for 7 years at 8%.
10. $ 763.70 for 9 years at 6%.
11. $ 544.50 for 5 years at 7%.
12. Find the interest of $ 672.40 for 3 years 9 months at 6%.

SOLUTION.

$672.40
.06
$40.3440 = Interest for 1 year.
3
$121.0320 = Interest for 3 years.
30.258 = Interest for 9 months ($\frac{3}{4}$ yr.) = $\frac{3}{4}$ of $40.3440.
$151.2900 = Interest for 3 years and 9 months.

Or

$672.40
.22½ = Rate for 3 years and 9 months, or 3¾ years.
33620
134480
134480
$151.2900 = Interest for 3 years and 9 months.

Find the interest of

13. $ 784.30 for 2 years 6 months at 8%.
14. $ 322.50 for 3 years 4 months at 6%.
15. $ 539.35 for 4 years 2 months at 8%.
16. $ 684.90 for 5 years 8 months at 9%.
17. $ 739.40 for 6 years 5 months at 6%.
18. $ 828.75 for 2 years 9 months at 8%.
19. $ 960.20 for 3 years 10 months at 7%.
20. $ 983.45 for 7 years 11 months at 12%.
21. $ 345.39 for 5 years 5 months at 5%.
22. $ 649.60 for 3 years 1 month at 7%.
23. $ 849.72 for 3 years 6 months at 4½%.
24. $ 227.30 for 4 years 8 months at 3½%.
25. $ 672.80 for 5 years 7 months at 5¾%.

315. 1. Find the interest of $ 640 for 5 yr. 8 mo. 25 da. at 6%.

SOLUTION.

$640
.06
12 | $38.40 × 5 = $192 = Interest for 5 yr.
6 | $ 3.20 × 8 = $ 25.60 = Interest for 8 mo.
.53⅓ × 5 = $ 2.67 = Interest for 25 da. or ⅚ mo.
$220.27 = Interest for 5 yr. 8 mo. 25 da.
640.
$860.27 = Amount.

NOTE. — If the mills are less than 5, disregard them; if more than 5, regard them as 1 cent.

Find the interest and amount of

2. $825.60 for 3 yr. 10 mo. 15 da. at 6%.
3. $936.72 for 4 yr. 9 mo. 18 da. at 8%.
4. $826.80 for 5 yr. 8 mo. 12 da. at 6%.
5. $673.20 for 6 yr. 7 mo. 21 da. at 7%.
6. $892.75 for 2 yr. 5 mo. 17 da. at 5%.
7. $560.15 for 3 yr. 3 mo. 3 da. @ 3%.
8. $760.90 for 4 yr. 2 mo. 5 da. @ 4½%.
9. $345.10 for 5 yr. 4 mo. 6 da. @ 5%.
10. $829.20 for 2 yr. 10 mo. 10 da. @ 10%.
11. $246.30 for 1 yr. 6 mo. 15 da. @ 8%.
12. $195.00 for 5 yr. 11 mo. 27 da. @ 9%.
13. $172.40 for 6 yr. 6 mo. 6 da. @ 6%.

316. When examples involve years, months, and days, the time may be reduced to the decimal of a year or of a month. In the example above, to find the interest of $825.60 for 3 yr. 10 mo. 15 da. at 6%, the time is equal to 46.5 months, or $\frac{1}{12}$ as many years = 3.875 years.

Whence the following

SOLUTION.

```
      $825.60
          .06
     $49.5360 = Interest for 1 year.
        3.875
      2476800
     3467520
    3962880
   1486080
 $191.9520000 = Interest for 3 yr. 10 mo. 15 da.
```

Or

```
      $825.60
         .00½
      $4.1280 = Interest for 1 mo.
         46.5
       206400
      247680
     165120
   $191.95200 = Interest for 3 yr. 10 mo. 15 da.
```

Six Per Cent Method.

317. The interest of any sum at 6%

For 1 year is .06 of itself.
For 1 month is $\frac{1}{12}$ of .06, or .005 of itself.
For 1 day is $\frac{1}{30}$ of .005, or $.000\frac{1}{6}$ of itself.

1. What is the interest of \$275 for 2 yr. 3 mo. 12 da. at 6%?

Solution.

Rate for 2 yr. = $2 \times .06 = .12$
Rate for 3 mo. = $3 \times .005 = .015$
Rate for 12 da. = $12 \times .000\frac{1}{6} = .002$
Rate for 2 yr. 3 mo. 12 da. = $.137$

```
 $275 = Principal.
  .137 = Rate.
  ----
  1925
  825
 275
 -------
$37.675 = Interest.
```

To find the interest at any other rate per cent, divide the interest at 6% by 6, and multiply the result by the given rate.

318. Find the interest and amount of

1. \$310 for 1 yr. 5 mo. 6 da. at 12%.
2. \$760.40 for 3 yr. 8 mo. 12 da. at 9%.
3. \$836.90 for 5 yr. 9 mo. 18 da. at 8%.
4. \$972.30 for 4 yr. 10 mo. 9 da. at 6%.
5. \$1895.75 for 2 yr. 7 mo. 15 da. at 4%.
6. \$2750.25 for 3 yr. 6 mo. 3 da. at 3%.
7. \$2863.40 for 1 yr. 3 mo. 21 da. at 7%.

ANNUAL INTEREST.

319. When a note stipulates "with interest payable annually," or "with interest annually," the interest is due at the end of each year, and is called annual interest.

The annual interest, when not paid, draws simple interest until paid.

Find the interest and amount of the following (the payment of interest being deferred to the final settlement):

1. \$600 for 3 yr. 6 mo. at 6%, annual interest.
2. \$1800 for 4 yr. 4 mo. at 8%, annual interest.
3. \$2400 for 2 yr. 8 mo. at 6%, annual interest.
4. \$3250 for 3 yr. 9 mo. at 7%, annual interest.
5. \$5678 for 5 yr. 6 mo. at 5%, annual interest.

II. COMPOUND INTEREST.

320. When the amount for each year, or period of time, beginning with the first, is made the principal for the next period, the interest thus accrued, which is found by subtracting the original principal from the final amount, is called Compound Interest.

1. Find the compound interest of \$500 for 3 years at 6%.

SOLUTION.

$$\begin{array}{rl}
\$500 & \\
1.06 & \\
\hline
3000 & \\
500 & \\
\hline
\$530.00 & = \text{Amount for 1st yr. ; principal for 2d yr.} \\
1.60 & \\
\hline
318000 & \\
53000 & \\
\hline
\$561.8000 & = \text{Amount for 2d yr. ; principal for 3d yr.}
\end{array}$$

$561.80 = Amount for 2d yr.; principal for 3d yr.
1.60

337080
56180

$595.5080 = Amount for 3d yr.
$500. = Original principal.

$95.508 = Compound interest for 3 years.

Find the compound interest of the following:

2. $ 540 for 2 yr. at 8%.
3. $ 720 for 3 yr. at 5%.
4. $ 840.75 for 4 yr. 6 mo. at 6%.
5. $ 960.40 for 3 yr. 4 mo. at 7%.
6. $ 1080.25 for 5 yr. at 6%.

III. PARTIAL PAYMENTS.

321. A Partial Payment is a payment in part of a note, bond, or other obligation.

UNITED STATES RULE.—*Find the amount of the principal to the time when the payment, or the sum of the payments, equals, or exceeds, the accrued interest, and from this amount subtract the payment, or the sum of the payments. The remainder is a new principal, with which proceed as before.*

1. $ 600. ATHENS, GA., Jan. 1, 1890.

Three years after date, I promise to pay Malcolm McCord, six hundred dollars, with interest from date, at 6 per cent, value received. JOHN JONES.

On this note were endorsed the following payments:

Jan. 1, 1891, $50.
Jan. 1, 1892, $30.

Find the amount when the note fell due.

Solution.

$600 = Principal.
1.06 = 1 plus rate.
3600
600
$636.00 = Amount at end of 1st year.
50.00 = First payment.
$586.00
1.12 = 1 plus rate for 2 yr. (payment being less than interest).
117200
58600
58600
$656.3200 = Amount at end of 3d year.
30.00 = Second Payment.
$626.32 = Amount due Jan. 1, 1893.

2. A note for $840, dated Jan. 12, 1892, was endorsed as follows: April 2, 1892, $75; Aug. 22, 1892, $50; Nov. 12, 1892, $205. Find the amount due Jan. 12, 1893, at 6%.

3. A note for $1500, dated Oct. 1, 1893, was endorsed as follows: Dec. 1, 1893, $225.50; March 16, 1894, $525.25; July 31, 1894, $375.25. Find balance due Oct. 1, 1894, at 8%.

4. A note for $350, dated Dec. 1, 1894, was endorsed as follows: Jan. 1, 1895, $100; Feb. 16, 1895, $150; March 1, 1895, $5.00; April 15, 1895, $45. Find the amount due May 1, 1895, at 7%.

5. A note for $1250, dated Feb. 1, 1892, was endorsed as follows: Feb. 1, 1893, $250; May 1, 1894, $375; Dec. 1, 1894, $280. What amount was due Feb. 1, 1895, at 6%?

MERCANTILE RULE.

322. Merchants often settle notes and accounts upon which payments have been made by the following:

RULE. — *Find the amount of the note, or account, from the time interest began to the time of final settlement.*

Find the amount of each payment from the time it was made to the time of final settlement; and from the amount of the principal subtract the sum of the amounts of the payments; the remainder will be the balance due.

1. On a note for $125, dated May 15, 1893, were endorsed the following payments: July 15, 1893, $25; Sept. 15, 1893, $30; Dec. 1, 1895, $40. What balance was due May 15, 1894, at 6%?

2. On an account of $87.50, bearing interest from Jan. 1, 1894, were endorsed the following payments: Jan. 16, 1894, $5.50; March 1, 1894, $7.50; April 15, 1894, $10; June 15, 1894, $12.50; Aug. 12, 1894, $8.25; Oct. 1, 1894, $25. Dec. 1, 1894, the account was paid in full. What was the amount of the final payment? Rate 7%.

IV. PRESENT WORTH AND DISCOUNT.

323. The **Present Worth** of a debt, due at some future time, is that sum which, placed at interest for the given time, at a given rate, will produce the debt when due.

324. **Discount** is the face value less the present worth of the debt.

NOTE. — This is often called true discount, to distinguish it from commercial discount, or per cent off, which is a deduction, from the face of an obligation, of a given per cent without regard to time.

325. When expressed in the terms of interest —

The Present Worth becomes the Principal.
The Discount becomes the Interest.
The Face of the Debt becomes the Amount.

1. What is the present worth and the discount of a note for $ 187.27, due in 1 yr. 4 mo., without interest, when money is worth 6% ?
2. Find the present worth of a note for $ 420.90, due in 2 yr. 9 mo., without interest, the current rate of interest being 8%. What is the discount?
3. What is the present worth and the discount of a note for $ 387.28, due in 6 mo., without interest, the current rate being 6% ?
4. Find the present worth and the discount of a note for $ 1491, due in 2 yr. 3 mo. 15 da., without interest, at 8%.

V. BANK DISCOUNT.

326. A Bank is an Institution, established by law, to lend money, to receive money on deposit, to issue drafts, bills of exchange, etc.

327. A **Promissory Note** is a written promise to pay a certain sum of money, at a specified time, in consideration of value received.

The law allows the maker of a note three days after the time specified in the note, to meet his obligation.

This extension is called **Days of Grace.**

328. **Bank Discount** is the simple interest upon the face of a note for the specified time plus the three days of grace.

The **Proceeds** of a note is the face of the note less the bank discount.

The **Term of Discount** is the number of days from the time a note is discounted to the time it matures.

329. Find the discount and proceeds of the following notes:

1. $ 360. ATLANTA, GA., May 22, 1895.

Three months after date, I promise to pay J. P. Davis three hundred sixty dollars for value received at the First National Bank. JOHN JONES.

Discounted June 22, 1895, at 6%.

SOLUTION.

The note matures Aug. 25, 1895.

The term of discount is from June 22 to Aug. 22, 1895 = 64 days.

The bank discount is the interest on $ 360 for 64 days, which at 6% = $ 3.84.

The proceeds = $ 360 − $ 3.84 = $ 356.16.

2. $ 250. AUGUSTA, GA., Jan. 1, 1895.

Sixty days after date, I promise to pay Jacob Haas two hundred fifty dollars for value received at the First National Bank. J. B. HUNNICUTT.

Discounted Feb. 1, 1895, at 8%.

3. $ 750.50. MACON, GA., Nov. 1, 1895.

Three months after date, I promise to pay J. Q. Waxelbaum seven hundred fifty and $\frac{50}{100}$ dollars for value received at First National Bank. J. S. PROUDFIT.

Discounted Dec. 1, 1895, at 6%.

4. $ 630. COLUMBUS, GA., May 1, 1895.

Three months after date, I promise to pay James Featherston six hundred thirty dollars for value received at First National Bank. J. P. BROWER.

Discounted June 1, 1895, at 6%.

5. $ 490. SAVANNAH, GA., March 1, 1895.

Two months after date, I promise to pay David Brown four hundred ninety dollars for value received at Second National Bank. THOMAS SMITH.

Discounted April 1, 1895, at 6%.

CHAPTER XIV.

INVOLUTION AND EVOLUTION.

330. **Involution** is the process of finding the power of a number.

331. The **Power** of a number is the product produced by taking the number one or more times as a factor.

332. The **Exponent** or **Index** is a number that denotes the degree of the power; as 5^3 indicates $5 \times 5 \times 5$, or the third power of 5.

The first power of $3 = 3^1$ or 3.
The second power or square of $3 = 3 \times 3 = 3^2$ or 9.
The third power or cube of $3 = 3 \times 3 \times 3 = 3^3$ or 27.
The fourth power of $3 = 3 \times 3 \times 3 \times 3 = 3^4$ or 81.
The fifth power of $3 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$ or 243.

333. **Evolution** is the process of finding a root of a number.

334. The **Root** of a number is one of the equal factors of the number.

335. The **Square Root** of a number is one of the *two* equal factors of the number.

336. The **Root** or **Radical Sign** is a character [$\sqrt{\ }$] placed before the number to denote the degree of the root; $\sqrt{25}$ denotes the square root of 25, $\sqrt[3]{64}$ denotes the cube root of 64. The root of a number may be obtained by resolving it into its equal factors.

337. Find the square root of 225, 784, and 324.

PROCESS.

3	225
3	75
5	25
5	5
	1

$225 = 15 \times 15$;
$\sqrt{225} = 15$.

PROCESS.

4	784
4	196
7	49
7	7
	1

$784 = 28 \times 28$;
$\sqrt{784} = 28$.

PROCESS.

9	324
9	36
2	4
2	2
	1

$324 = 18 \times 18$;
$\sqrt{324} = 18$.

338. Find the square root of 625, of 676, and the cube root of 1728.

PROCESS

5	625
5	125
5	25
5	5
	1

$625 = 5^2 \times 5^2$ or 25×25;
$\sqrt{625} = 25$.

PROCESS.

2	676
2	338
13	169
13	13
	1

$676 = 26 \times 26$;
$\sqrt{676} = 26$.

PROCESS.

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$1728 = 12 \times 12 \times 12$;
$\sqrt[3]{1728} = 12$.

It is tedious and difficult and sometimes impossible to resolve large numbers into their equal factors; for this reason, a different method is commonly used. This method is however essentially algebraic, and students at this stage of advancement find it impossible thoroughly to master its rationale.

In this treatise no attempt is therefore made to demonstrate the rules of Evolution; but, instead, simple geometrical illustrations are given as helps to fix the processes in the mind of the pupil.

SQUARE ROOT.

339. The second power has twice as many orders of units as the root, or twice as many less one.

$1^2 = 1.$ $10^2 = 100.$ $100^2 = 10000.$
$3^2 = 9.$ $25^2 = 625.$ $1000^2 = 1000000.$
$9^2 = 81.$ $99^2 = 8801.$ $2.5^2 = 6.25.$

Hence, to find the number of figures in the root, separate the number into periods of two figures each, beginning at the place of the decimal point. The left-hand period will contain the square of the first figure of the root.

340. The floor of a square hall contains 5625 sq. ft.; find the length of each side of the square.

PROCESS.

$$\begin{array}{r|l} \dot{5}6\dot{2}5 & 70 + 5 = 75 \\ 4900 & \\ \hline 145)\ 725 & \\ 725 & \\ \hline \end{array}$$

SHORTENED PROCESS.

$$\begin{array}{r|l} \dot{5}6\dot{2}5 & 75 \\ 49 & \\ \hline 145)\ 725 & \\ 725 & \\ \hline \end{array}$$

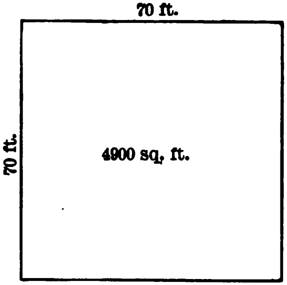


FIG. 1.

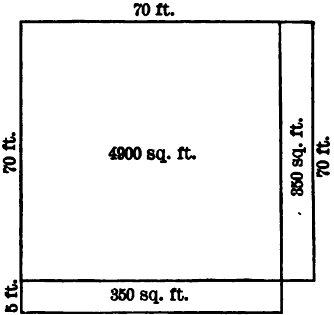


FIG. 2.

EXPLANATION. — The floor is a square whose area is equal to the product of its length by its breadth; hence the square root of the area, 5625 sq. ft., will give the length of each side of the square.

To find the number of figures in the root, we separate the number into periods of two figures each, beginning at the place of the decimal point, and find that there are two orders of units in the root.

We extract the square root of the left-hand period and find it to be 7(0) ft. Subtracting the square of the root found, which is 49(00) sq. ft., from the area of the hall, 5625 sq. ft., we have a remainder of 725 sq. ft.

That the form of the square may be preserved, we must add this remainder equally to the length and breadth of the square (Fig. 2).

Since the length of each side of the square found is 70 ft., the length of the entire addition is twice 70 ft., or 140 ft. plus the length of the small square (Fig. 3), whose length is the breadth of the addition which is as yet undetermined.

The breadth of the added area being small in comparison with its length, 140 ft. may be regarded as its approximate length; and the added area divided by this approximate length as a trial divisor will give its breadth (nearly).

Dividing 725 sq. ft. by 140 ft., we get 5 (nearly). If 5 ft. is the exact breadth of the added area, we find its exact length by adding 5 ft. to the approximate length, which gives 145 ft. (Fig. 4.)

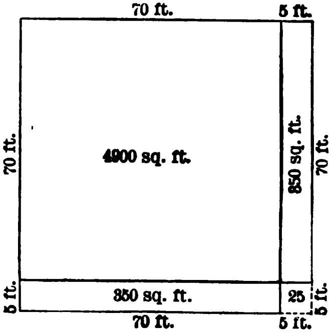


Fig. 8.

Since the length multiplied by the breadth produces the area, we multiply 145 ft. (the length) by 5 ft. (the breadth), and find that the product equals the remainder, 725 sq. ft. Whence we conclude that the remainder has been properly distributed and the side of the square found, which is 75 ft.

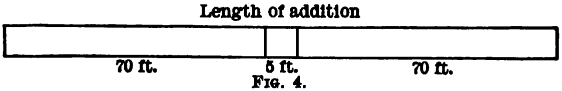


Fig. 4.

Rule.—*Begin at the place of the decimal point and separate the number both ways into periods of two figures each.*

Find the greatest square in the left-hand period, and place its square root in the form of a quotient at the right of the number.

Subtract the square of this root figure from the left-hand period, and to the remainder annex the next period for a dividend.

Take twice the figure of the root found for a trial divisor; divide the dividend, omitting the last figure, by this trial divisor, and annex the quotient thus found to the root and also to the divisor. Multiply the complete divisor by the figure of the root last found, and subtract the product from the dividend.

Proceed in the same way with the remaining period (if any) until all are brought down.

Point off one-half as many decimal places in the root as there are decimal places in the power.

341. The square root of a common fraction is found by taking the square root of the numerator and of the denominator for the terms of the root.

Find the square root of

1. 676.
2. 729.
3. 784.
4. 841.
5. 1369.
6. 1521.
7. 2209.
8. 4624.
9. 5625.
10. 7921.
11. 8649.
12. 15625.

Cube Root.

342. The third power has three times as many orders of units as its root, or three times as many less one or less two.

$1^3 = 1.$

$2^3 = 8.$

$3^3 = 27.$

$9^3 = 729.$

$10^3 = 1000.$

$99^3 = 970299.$

$100^3 = 1000000.$

$1000^3 = 1000000000.$

Hence there will be as many orders, or figures, in the cube root of a number as there are periods of three figures each in the number.

343. A cubical barn has a volume of 421875 cu. ft.; find its dimensions.

PROCESS.

$$\begin{array}{lrr|l} & & 42\dot{1}87\dot{5} & \underline{70+5} \\ 70^3 = & & \underline{343000} & \\ & 14700 & \boxed{78875} & \\ & 1050 & \underline{78875} & \\ & \underline{25} & & \\ & 15775 & & \end{array}$$

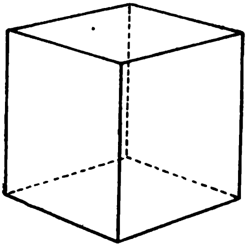


FIG. 1.

EXPLANATION.—The barn is a perfect cube whose volume is equal to the product of its length, breadth, and thickness; hence the cube root of its volume will be the length of each of its dimensions.

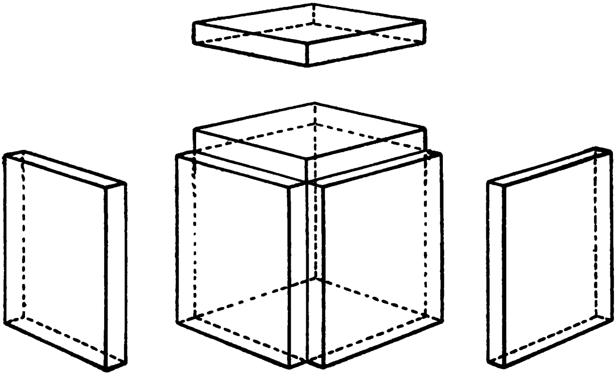


FIG. 2.

To find the number of figures in the root, we separate the number into periods of three figures each, beginning at the place of the decimal point, and find that there are two orders of units in the root.

We extract the cube root of the left-hand period and find it to be 7(0) ft. Subtracting the cube of the root found (Fig. 1), which is 343(000) cu. ft., from the volume of the barn, 421,875 cu. ft., we have a remainder of 78,875 cu. ft.

That the form of the cube may be preserved, we must add the remainder equally to the length, breadth, and thickness of the small cube (Fig. 1).

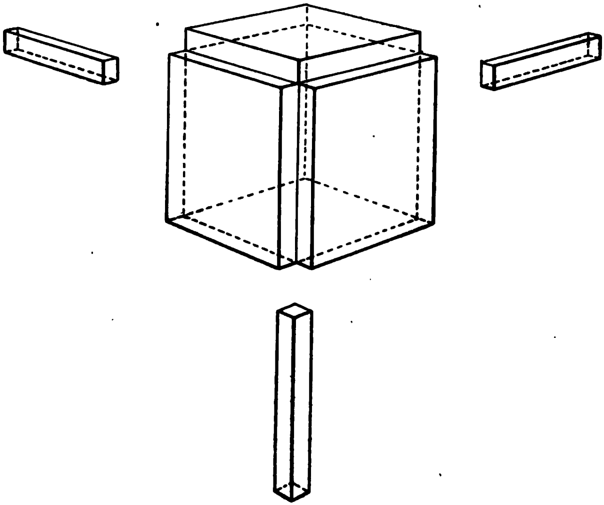


FIG. 3.

The remainder is added in three instalments: 1st, 3 solid layers are added (Fig. 2); 2d, 3 solids (Fig. 3); 3d, the minute cube (Fig. 4).

Since each of the layers covers a face of the cube (Fig. 2), the area of each layer must be that of one of the faces of the cube; namely, 70 ft. × 70 ft., or 4900 sq. ft., and the area of the three, 14,700 sq. ft. The thickness of the layer is as yet undetermined.

The length of each solid addition (Fig. 3) is the length of one edge of the cube (Fig. 1), which is 70 ft.; and its breadth and thickness are equal, and each that of the undetermined thickness of the layers.

The dimensions of the minute cubical addition (Fig. 4) are each that of the undetermined thickness of the layers.

The combined volume of the additions divided by their combined area will give their common thickness; but we have only one dimension of the long solids and none of the minute cube.

But the area of the solids and minute cube being small in comparison with that of the three layers, the area of the latter, 14700 sq. ft., may be regarded as the approximate area of the additions; and the added volume divided by this approximate area as a trial divisor will give the common thickness of the several additions (nearly).

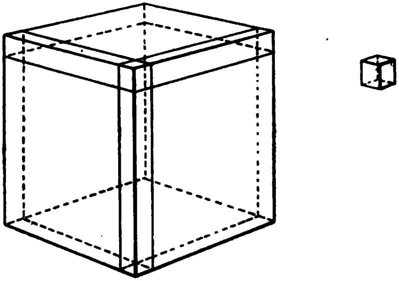


FIG. 4.

Dividing the remaining volume, 78875 cu. ft., by the approximate area of the additions, 14700 sq. ft., we obtain 5 ft. as the thickness of the additions.

We place 5 in the root and proceed to complete the divisor.

Area of 3 layers (each 4900)	14700 sq. ft.
Area of 3 solids (70 × 5 = 350)	1050 sq. ft.
Area of 1 minute cube (5 × 5 = 25) . .	25 sq. ft.
Combined area of additions	15775 sq. ft.

The complete divisor, 15775 sq. ft., multiplied by the second figure of the root (5 ft.) will give 78875 cu. ft., which is the volume of the additions. Whence we conclude the remainder has been properly distributed and the edge of the cube found, which is 75 ft.

RULE. — *Begin at the place of the decimal point, and separate the number both ways into periods of three figures each.*

Extract the cube root of the left-hand period, and place the figure found to the right of the number in the form of a quotient, as the first term of the root; cube this figure regarded as tens, and subtract the product from the left-hand period; and to the remainder bring down the next period for a dividend.

Take three times the square of the figure found, regarded as tens, for a trial divisor, by which divide the dividend, and place the quotient in the root. To the trial divisor add three times the product of the second figure of the root by the first, regarded as tens, and also the square of the second figure. The sum will be the complete divisor, which multiply by the second figure of the root and subtract the product from the dividend.

Proceed in the same way with the remaining periods (if any) until all are brought down.

Point off one-third as many decimal places in the root as there are decimals in the power.

Find the cube root of

1. 15625.
2. 132651.
3. 238328.
4. 117649.
5. 405224.
6. 636056.
7. 941192.
8. 1953125.
9. 13312053.
10. 42508549.
11. 92345408.
12. 178453547.
13. 306546875.
14. 731432701.

The cube root of a common fraction is found by taking the cube root of the numerator and of the denominator for the terms of the root.

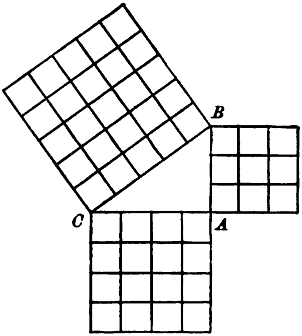
Applications of the Square and Cube Roots.

1. Find the side of a square whose area is 49.4209 square yards.
2. Find the edge of a cube whose volume is 212776173 cubic yards.
3. Required the area of each of the faces of a cube whose volume is 531441 cubic inches.

4. How much tin will it take to make a cubical box (without cover) whose contents shall be 357911 cubic inches?

5. A triangle is a plane bounded by three straight lines and having three angles. A right-angled triangle is a triangle having one right angle.

6. The square described upon the hypotenuse (side opposite the right angle) of a right-angled triangle is equivalent to the sum of the squares described upon the other two sides: whence, the hypotenuse is equal to the square root of the sum of the squares of the other two sides; and the base or perpendicular is equal to the square root of the difference of the squares of the hypotenuse and the other side.



7. The base and perpendicular of a right-angled triangle are respectively 8 feet and 6 feet. Required the hypotenuse.

8. The hypotenuse of a right-angled triangle is 5 feet and its perpendicular, 4 feet. Find its base.

9. The sides of a right-angled triangle are each 1 foot. Required the length of the hypotenuse.

10. The floor of a room is 16 feet by 20 feet: find the distance between the opposite corners.

11. A ladder 50 feet long is placed on the ground 30 feet from the basement of a house and just reaches the lower sill of the window: find the height of the window.

MISCELLANEOUS EXAMPLES IN REVIEW.

1. If the minuend is 39672 and the difference 26745, what will the subtrahend be?

2. The divisor is 54321 and the quotient 12345. Find the dividend.

3. The dividend is 45780840 and the quotient 555. Find the divisor.

4. The dividend is 34705, the quotient 45, and the remainder 505. Find the divisor.

5. A shepherd wishes to put 693 goats and 1875 sheep in the least number of pens that will hold the same number of head. How many head must each pen contain?

6. A earns $ 57 a month, B $ 95 a month, and C $ 133 a month. Find the least sum of money that will furnish a number of integral months' work to each of the men.

7. Nellie had $\frac{17}{23}$ of a melon and Harry gave her $\frac{53}{90}$ of an equal melon. What part of the melon did she then have?

8. Leila spent $\frac{27}{35}$ of a certain sum of money for ribbon, giving $\frac{3}{5}$ of the original sum for each yard. How many yards did she buy?

9. What will $16\frac{2}{3}$ yards of cloth cost at $16\frac{2}{3}$ cents a yard?

10. A merchant owning a $\frac{3}{4}$ interest in a business sells $\frac{1}{3}$ of his interest. What part of the business does he still own?

11. If 75 calves cost $ 656.25, how many calves can be bought with $ 700?

12. What will 36 yards of cloth bring at the rate of 75¾ yards for $ 643.87½ ?

13. If $\frac{4}{7}$ of a store be worth 943£ 7*s.* 8*d.*, what is the value of the store in dollars?

14. Washington, D.C., is 77° 0′ 15″ west longitude and San Francisco 122° 26′ 48″ west longitude. When it is noon at Washington, what time is it at San Francisco?

15. If a man travels 281.826 miles in 4.59 days, how far does he travel in 7.75 days?

16. The paymaster of the Seaboard Air Line distributed $ 26137.65 equally among a certain number of men, giving each $71.61. How many men were there?

17. $ 58.59 will buy how many yards of cloth at $ .93 a yard?

18. How many gallons of syrup may be bought for $ 176.25 at $ .47 a gallon?

19. If a clerk has for the past 15 years smoked 5 cigars a day at an average cost of 3½ cents each, how much has this vicious habit cost him, reckoning the year at 365 days?

20. Find the least common multiple of 620, 558, and 744.

21. What is the L. C. M. of 6942 and 3738?

22. What number multiplied by 7 thousandths gives 29.925?

23. How many slabs of marble 1 foot long and 8 inches wide, will it take to pave a court 100 yards square?

24. What is the difference between 100 yards square and 100 square yards?

25. How many bushels of corn will a barn 50 feet long, 30 feet wide, and 25 feet high hold?

26. What will it cost to excavate a cellar 40 feet long, 30 feet wide, and 10 feet deep at 50 cents a cubic yard?

27. How many gallons of water can be held by a ditch 9 miles long, 3 feet wide, and 5 feet deep?

28. How many cubic feet in a syrup boiler that holds 60 gallons?

29. Which is worth the more, 77 bushels of wheat at $ $1.37\frac{1}{2}$ a bushel, or 118 bushels of corn at $ .89 a bushel?

30. A boy studies 5 hours a day (Sundays excepted) from Feb. 1 to June 20. How many days of 12 hours each did he employ in study?

31. If a person sleeps 8 hours a day, how many years will he spend in sleep in a life of 3 score and 10 years?

32. From the product of the sum and difference of 4.6 and 2.5 take the difference of their squares.

33. A merchant paid $ 1300 for 26 bales of merchandise. At the same rate what would 178 bales cost?

34. What will 45 yards of cloth cost at the rate of 33 yards for $ 7.26?

35. Find the cost of $8\frac{6}{7}$ pounds of sugar at the rate of $5\frac{2}{3}$ pounds for 42 cents.

36. At the rate of $3\frac{1}{2}$ yards of silk for $ 3, what will $28\frac{2}{7}$ yards cost?

37. Find the cost of $\frac{11}{20}$ of a pound of cheese at the rate of $\frac{11}{15}$ of a pound for $\frac{8}{15}$ of a dollar.

38. 6 yards of flannel were sold for $ $4.55\frac{35}{59}$. At the same rate, what would $7\frac{3}{8}$ yards have brought?

39. Find the cost of $16\frac{2}{3}$ yards of cloth at the rate of $12\frac{1}{2}$ yards for $ 1.

40. If a 3-foot staff casts a 12-foot shadow, what length of shadow will a 9-foot staff cast at the same time of day?

41. At the rate of 2 ounces of silver for $2.24, what will $\frac{3}{8}$ of an ounce bring?

42. Find the cost of 94 barrels of flour at the rate of $425 for 100 barrels.

43. What 4 men do in 100 days, 5 men would be how long in doing?

44. 2 men would be how long in doing what 20 men do in $\frac{1}{2}$ day?

45. How many men would do in 20 days what 60 men do in 8 days?

46. How many days would it take 10 men to plough a field that it takes 2 men 40 days to plough?

47. If $\frac{3}{7}$ of a book equals 120 pages, how many pages does the book contain?

48. A can do a piece of work in 21 hours, and B can do the same work in 28 hours. How long will it take A and B working together to do the work?

49. A can do a piece of work in 30 days, B in 45 days, and C in 180 days. In what time can A and B do the work? In what time A and C? In what time B and C? In what time A, B, and C?

50. A and B can do a piece of work in 40 days; A and C in 15 days; and B and C in 24 days. In what time can A, B, and C together do the work?

51. What number added to $\frac{1}{2}$ of itself equals 1833?

52. What number increased by $\frac{1}{2}$ and $\frac{1}{8}$ of itself equals 548?

53. A stock breeder has 2888 head of horses, cows, and sheep, and he has 3 times as many cows as horses and 5 times as many sheep as cows. How many of each has he?

54. Two candidates for the office of county school commissioner receive 781 votes, and the successful candidate gets 51 votes more than the other. How many votes does each get?

55. A has 180 yards the start of B, who gains 5 yards every minute. In what time will B overtake A?

56. If a miller takes $12\frac{1}{2}\%$ of the grain he grinds as toll, how many bushels of grain must a person take to mill that he may get 28 bushels of meal?

57. Divide $ 2860 among 3 persons so that the second shall have $ 50 more than the first, and the third $ 60 more than the second.

58. What is 7520 tons of hay worth at $ 1.25 a hundred pounds?

59. Divide 25 into 2 parts that shall have the ratio to each other of 3 to 2.

60. Divide 27 into 3 parts that shall have the ratio to each other of 2, 3, 4.

61. What is the height of a mountain on whose summit the boiling point of water is 180° Fahrenheit?

NOTE. — The boiling point of water at the sea level is 212°, and the temperature falls one degree for every 553 feet of ascent.

62. A person owning $\frac{3}{5}$ of a mill sells $\frac{2}{3}$ of his share for $ 400. Required the worth of the mill.

63. If 18 men in 16 weeks plough a farm of 1152 acres, how many acres can 15 men plough in 18 weeks?

64. If 16 men in 2 weeks earn $ 720, how much will 11 men earn in 9 months?

65. If 10 horses eat 8 tons of hay in 6 weeks, how many horses will it take to eat 56 tons in 21 weeks?

66. If 6 men earn $ 84 in 14 days, what will 9 men earn in 16 days?

67. If 6 laborers dig a ditch 34 rods long in 10 days, how many rods will 20 laborers dig in 15 days?

68. If 96 cows eat $3\frac{3}{4}$ tons of hay in $7\frac{1}{2}$ weeks, how much hay will last 32 cows for 120 weeks?

69. A farmer sold his farm for $ 3360, gaining thereby 20% of the cost. Required the cost.

70. A real estate dealer sold a house and lot for $ 1800, losing thereby 20% of the cost. Find the cost.

71. A house and lot together are worth $ 8250, and the house is worth 10 times as much as the lot. Find the value of each.

72. A and B entered into copartnership, A furnishing $ 5470, and B $ 6580; their net gain the first year was $ 843.50. Find the share of each.

73. What is the interest on a note of $ 560 for 3 years, 8 months, and 15 days?

74. For what must 6% stock be bought that the investment share yield an income of 8%?

75. A man sold two horses for $ 150 each, on the one he gained 25%, and on the other he lost 25%. Did he gain or lose on the sale, and how much?

76. A citizen's assessed property is $ 12800, and the annual tax for the support of public schools is 25 cents on the hundred dollars. What does he contribute to the support of the schools?

77. A piece of property is bought for $ 27250 and sold for $ 32018.75. What per cent is gained?

78. Find the interest of $ 371.25 for 3 years, 9 months, 12 days, at 4%.

79. A shepherd lost 8% of his sheep by disease, 33⅓% of the remainder were sold, and there still remained 518 sheep. What number had he at first?

80. How much must I ask for a horse that cost $ 122.50, that I may deduct 30% from my asking price and still make 30%?

81. If stock is bought at 8% below par and sold at 6% above par, how many dollars will be made on 75 shares?

82. A horse and buggy together are worth $ 300, and the horse is worth 50% more than the buggy. Required the cost of each.

83. Mr. Thomas Nall of Griffin, Ga., received $ 7500 to invest in cotton, after deducting his commission of 2%. Find his commission.

84. A commission merchant receives $ 1000 to invest in bacon, after deducting his commission of 5%. Required the amount invested in bacon and his commission.

85. If 100 shares of railroad stock be bought at par and sold at 3% discount, what is the loss?

86. 40 yards of cloth bought at $ 2.50 per yard and sold for $ 120 yields what rate per cent of profit?

87. A merchant failing in business owes *A* $ 500, *B* $ 1200, *C* $ 4300; and his assets are $ 1500. Find the amount each creditor receives.

88. Find the amount of a note of $ 525 from May 17, 1893, to June 18, 1895.

89. What principal at 8% interest from January 1, 1895 to March 16, 1895 will amount to $ 457.50 ?

90. A merchant having sold a quantity of goods for $ 600 on a credit of 1 year and 9 months, learned that he could have sold the same goods for $ 580 cash. Did he gain or lose by the transaction and how much, money being worth 6% ?

91. A boat left St. Louis for New Orleans with a cargo of 500 bales of cotton. The boat reached her destination with only 360 bales, the remainder having been thrown overboard. Required the per cent of cargo cast overboard.

92. Of 1250 boxes of oranges shipped from Florida to New York only 1075 boxes reached their destination in good condition. What per cent of the cargo had spoiled on the way ?

93. Mr. Hasselkus shipped 360 crates of the Alberta peach from Griffin, Ga., to Cincinnati; of these 270 crates were delivered to the consignee in good condition. What per cent loss did he sustain ?

94. What is the annual premium on a house worth $ 4500, the rate being $\frac{1}{4}$% ?

95. When the interest of $ 750 for 3 years and 4 months is $ 162.50, what is the rate per cent ?

96. Find the present worth and discount of a note for $ 1800, due in 1 year and 3 months, without interest, money being worth 6%.

97. What is the compound interest of $ 500 for 9 years, at 6% ?

98. Find the side of a square that contains 152399025 square yards.

APPENDIX.

MENSURATION.

1. **Mensuration** is the process of measuring lines, planes, and solids.

2. A **line** is length without breadth or thickness.

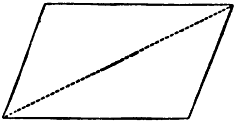
3. A **plane** is a surface having length and breadth without thickness.

4. A **solid** is a body having length, breadth, and thickness.

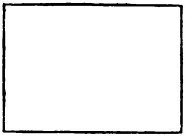
5. A **parallelogram** is a plane bounded by four straight lines with its opposite sides equal and parallel.

6. A **rectangle** is a parallelogram whose angles are right angles.

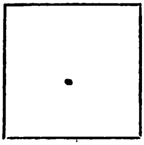
7. A **square** is a rectangle whose sides are equal.



Parallelogram.



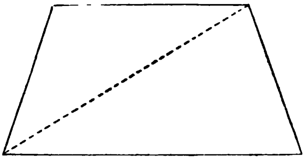
Rectangle.



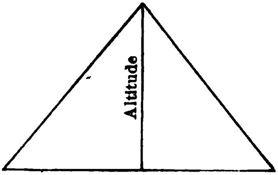
Square.

8. A **trapezoid** is a four-sided plane, two of whose sides are parallel.

9. The **altitude** of a triangle is the perpendicular distance between the vertex and base.



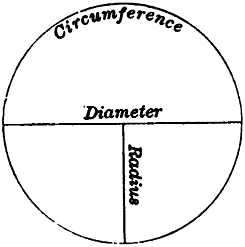
Trapezoid.



Triangle.

10. The **altitude** of a four-sided plane is the perpendicular distance between its upper and lower bases.

11. The **perimeter** of a plane (other than the circle) is the sum of the lengths of all its sides.



Circle.

12. A **circle** is a plane bounded by a curve line, every point of which is equally distant from a point within called the centre.

13. The **circumference** of a circle is the curve which bounds it.

14. The **diameter** is a straight line passing through the centre and terminating in opposite points in the circumference.

15. The **radius** is a straight line drawn from the centre to the circumference. It is one-half the diameter.

MEASUREMENT OF PLANES.

16. It is demonstrated in Geometry:

I. *That the area of a parallelogram is equal to the product of its base and altitude.*

II. *That the area of a triangle is equal to one-half the product of its base by its altitude.*

III. *That the area of a trapezoid is equal to the product of one-half the sum of its parallel sides by its altitude.*

IV. *That the circumference of a circle is equal to the product of its diameter by* 3.1416 (*nearly*).

V. *That the area of a circle is equal to the product of its radius by one-half the circumference.*

NOTE. — The area of a circle may also be found by multiplying the square of its diameter by .7854, or the square of its radius by 3.1416. Letting π represent 3.1416, R the radius, and A the area, we derive $\pi R^2 = A$, whence $R = \sqrt{\frac{A}{\pi}}$.

Examples.

1. How many square feet in a floor 20 feet long and 16 feet wide?
2. How many acres in a square piece of land whose sides are each 75 rods?
3. Required the area of a circle whose diameter is 25 rods.
4. What is the area of a triangle whose base is 40 feet and altitude 12 feet?
5. What is the area of a triangle whose base and altitude are respectively 160 feet and 75 feet?
6. Find the number of acres in a triangular field whose base is 250 rods and altitude 80 rods.
7. How many acres in a rectangular field 130 rods long and 90 rods wide?
8. Required the number of acres in a rectangular field 900 rods long and 160 rods wide.
9. 320 acres is the area of a rectangular field and 1 mile the length of one dimension. Find the length of the other dimension.

10. Find the area of a trapezoid whose parallel sides are respectively 60 rods and 80 rods, and its altitude 35 rods.

11. Find the area of a circle whose radius is 10 rods.

12. Find the area of a triangle whose base is 20 rods and altitude 6 rods.

13. Required the diameter of a circle whose circumference is 75 rods.

14. What is the area of a circle whose radius is 30 feet?

15. What is the area of a circle whose radius is 50 feet?

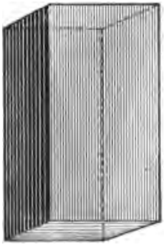
16. Find the circumference of a wheel whose diameter is 15 feet.

Measurement of Solids.

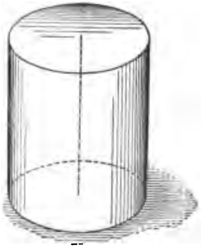
A **prism** is a solid described by the perpendicular movement of a plane.

A **cylinder** is a solid described by the revolution of a rectangle about one of its sides as an axis.

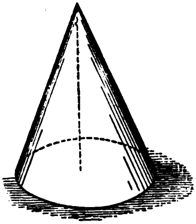
A **cone** is a solid described by the revolution of a right-angled triangle about its base or perpendicular as an axis.



Prism.



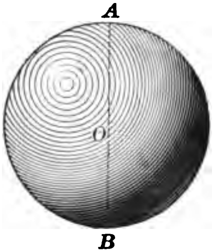
Cylinder.



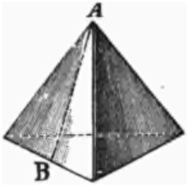
Cone.

A **sphere** is a solid described by the revolution of a semicircle about its diameter as an axis.

A **pyramid** is a solid having any regular plane (other than the circle) for its base, and three or more triangles for its sides, whose vertices meet in a common point called the vertex of the pyramid.



Sphere.



Pyramid.

It is demonstrated in geometry:

I. *That the area of the surface of a prism is the sum of the areas of its six faces.*

II. *That the area of the convex surface of a cylinder is equal to the product of the circumference of its base by its altitude.*

III. *That the area of the convex surface of a cone is equal to the product of the circumference of its base by one-half its slant height.*

IV. *That the area of a surface of a sphere is equal to the product of its diameter by the circumference of a great circle.*

V. *That the area of the lateral surface of a pyramid is equal to the product of the perimeter of its base by one-half its slant height.*

VI. *That the volume of a prism is equal to the product of its three dimensions (expressed in units of the same denomination).*

VII. *That the volume of a cylinder is equal to the product of the area of its base by its altitude.*

VIII. *That the volume of a cone is equal to the product of the area of its base by one-third of its altitude.*

IX. *That the volume of a sphere is equal to the product of the area of its surface by one-third of its radius.*

X. *That the volume of a pyramid is equal to the product of the area of its base by one-third of its altitude.*

Examples.

1. Find the surface of a sphere whose diameter is 18 feet.
2. Required the convex surface of a cone whose base is 25 square feet and altitude 18 feet.
3. Required the lateral surface of a pyramid, the perimeter of whose base is 30 rods and its slant height 42 yards.
4. What is the area of the convex surface of a cylinder, the circumference of whose base is 60 feet and altitude 12 feet?
5. Find the volume of a prism whose base is 25 square feet and altitude 10 feet.
6. Find the volume of a prism whose base is 25 feet square and altitude 10 feet.
7. Find the contents of a cone whose base is 36 square feet and altitude 9 feet.
8. Required the superficial area and the volume of a prism whose length, breadth, and thickness are respectively 7, 8, and 9 inches.
9. How many bushels in a box 15 feet long, 5 feet wide, and 8 feet deep?
10. What is the volume in cubic feet of a wall 24 feet 3 inches long, 10 feet 9 inches high, and 2 feet thick?

11. Find the superficial area and the volume of a sphere 24 inches in diameter.

12. What is the area of the surface of a prism whose altitude is 16 feet and base a square, each side of which is 5 feet?

13. Find the area of the lateral surface of a triangular pyramid whose slant height is 10 feet and the sides of its base 4, 5, and 6 feet.

14. Find the convex surface and volume of a cylinder whose circumference is 20 feet and altitude 45 feet.

15. Required the convex surface and volume of a cone, the circumference of whose base is 18.8496 feet and slant height 10 feet.

CARPETING ROOMS.

1. A floor is 15 feet by 12 feet. What will it cost to carpet it, each strip being $\frac{3}{4}$ yards wide, at $1.25 a yard?

SOLUTION. — (1) If the strips are laid lengthwise the room, it will take as many strips as $\frac{3}{4}$ yard is contained times in 12 feet, which is $5\frac{1}{3}$. Therefore it will require $5\frac{1}{3}$ strips; but since parts of a strip are never sold, 6 strips must be bought. Each strip must be the length of the floor, which is 15 feet, or 5 yards. That is, it will take 6 strips, each strip 5 yards long, or 30 yards of carpeting. Hence carpeting the room will cost 30 times $1.25 = $37.50.

(2) If the strips are laid crosswise the floor, it will take 7 strips ($6\frac{2}{3}$), each strip 4 yards long, or 28 yards of carpeting. 28 yards at $1.25 = $35.

2. How many yards of carpeting 1 yard wide will it take to cover a floor 18 feet by 15 feet?

3. Required the number of yards of matting 1 yard wide that will be necessary to cover a floor 20 feet by 16 feet.

4. Which is the cheaper way to lay the strips to cover a floor 18 feet by 15 feet with strips $\frac{5}{8}$ yards wide?

5. A room is 15 feet square. How many yards of carpeting will be required to cover it, if there is a waste of 3 inches in each strip in matching the pattern?

PAPERING WALLS.

Wall paper is usually sold in rolls 8 yards long and 18 inches ($\frac{1}{2}$ yard) wide. In papering walls the rolls are laid vertically. Since the rolls are $\frac{1}{2}$ yard wide, it will take twice as many widths of the roll to go round the room as there are yards in the perimeter of the floor. But a roll vertically placed will make as many strips as the height of the room is contained times in the length of the roll. With respect to allowance for doors and windows, custom varies. Allowance is sometimes made for one-half, sometimes for the whole area of doors and windows. A generous allowance is occasionally made by subtracting the width of the doors and windows from perimeter of floor.

1. How many rolls of paper of regular length and width will be required to paper the lateral walls of a room 16 feet long, 12 feet wide, and 9 feet high, making no allowance for doors and windows?

2. How much will it cost to paper a room 15 feet square and 9 feet high, at \$1.12$\frac{1}{2}$ a roll, making no allowance for doors and windows?

3. How many rolls of paper will it require to cover the lateral walls of a room 20 feet long, 16 feet wide, and 8$\frac{1}{2}$ feet high, making no deduction for apertures?

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